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THE MATHEMATICS TEACHER

Volume XXXVII NOVEMBER · 1944 Number 7

CONTENTS

		PAGE
Influence of the War on the Teaching of Secondary Mathematics	E. R. Breslich	291
The Hyperbolic Analogues of Three Theorems on the Circle	Joseph B. Reynolds	301
The Latent Proportional Parts Table in Ordinary Interpolation	Wm. Fitch Cheney, Jr.	304
Let Us Be Sensible About It	Irwin A. Buell	306
Schools and Colleges Face a Challenge	J. M. Bledsoe	309
Pitfalls in the Trigonometric Solution of an Oblique Triangle	Irwin M. Rothman	311
The Art of Teaching Functioning Mathematics	Samuel I. Altwerger	314
In Other Periodicals		323
News Notes		324
Official Notice		328

OFFICIAL JOURNAL PUBLISHED BY THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
MENASHA, WISCONSIN : NEW YORK 27, N.Y.

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of Mathematics in Elementary and Secondary Schools

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Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to the office of

THE MATHEMATICS TEACHER
525 West 120th St., New York 27, N.Y. (Editorial Office)

Subscription to THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE $2.00 PER YEAR (*eight numbers*)

Foreign Postage 50 cents per year; Canadian postage, 25 cents per year. Single copies 30 cents. Remittances should be made by Post Office Money Order, Express Order, Bank Draft or personal check made payable to THE MATHEMATICS TEACHER.

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THE MATHEMATICS TEACHER

Volume XXXVII Number 7

Edited by William David Reeve

Influence of the War on the Teaching of Secondary Mathematics

By E. R. Breslich
University of Chicago

Planning of a Post-war Curriculum Should Not Be Delayed

Adequate preparations for post-war conditions are being made before the war ends. For months statesmen and governments have been at work, searching for ways and means of securing a peace that will be lasting. Industries are making preparations for converting plants from war to peacetime production. Investors are weighing the possibilities for future safe and profitable investments for their funds. Questions of unemployment, transportation, business, housing, food and supplies are being studied. There is hardly a field in which problems are not waiting for solution.

In the midst of all these activities men and women interested in education have not been idle. They are preparing to meet the new educational problems of the near future. Committees have been appointed and are now at work. Plans for the future will be influenced by experiences during the war. Valuable lessons have been learned from the mathematics programs of preinduction and special training courses. They will influence the mathematical curriculum. The "Commission on Post-War Problems" of the *National Council of Teachers of Mathematics* has already published its first report.[1]

Previous to the War the Values of Mathematics Were Underrated

In many ways the attitudes toward mathematics before and during this war resemble those of World War I. In the years preceding each war high school mathematics received much adverse criticism from general educators, from teachers of higher mathematics and of related fields such as physics, from business and industry, and, indeed, from the parents of pupils. Many of the educational values claimed for the subject were being questioned and denied. The trend was to discourage pupils from electing the subject. An endless number of schools, ever so many school systems and an increasing number of states reduced or eliminated mathematical requirements for graduation.

The teachers of mathematics were not indifferent to these trends. Ample evidence is available in published articles and reports of the various national committees

[1] The First Report of the Commission on Post-War Plans," The Mathematics Teacher, XXXVII (May, 1944), 226–232.

that efforts were made to improve curriculum and methods, and to demonstrate through convincing argument the important role of mathematics in general education. Furthermore, many studies have disclosed the mathematical materials used in other school subjects and in the homes and daily lives of the people in order that they may receive proper emphasis in teaching. The mathematical curriculum has been correlated with various school subjects, especially the sciences, thereby making the subject real and training the pupils to use mathematics in other fields.

Moreover, the teachers of mathematics have contributed widely to the great educational movements, such as creating techniques for providing for individual differences in instruction; developing effective teaching procedures; training pupils in ways of studying; and organizing remedial programs to aid the slow pupils. Mathematics has been related to the purpose of general education and to the needs of society.

War Has Shown the Fallacy of Eliminating Mathematical Requirements

When war comes the importance of mathematical training is immediately recognized. This was true to a far greater extent in the present war than in World War I. At the very beginning ever so many men ready and eager to enlist had to defer applications for months until they fulfilled certain mathematical requirements. Rush courses in algebra, statistics, trigonometry and analytics were organized in the colleges and correspondence schools of the country to meet the needs of these students. Not only the armed forces, but also the great war industries and governmental statistical agencies were searching for men and women with sufficient mathematical background. Most skilled workers seemed to need some knowledge of high school mathematics and many good students were kept out of responsible positions as inspectors or as supervisors because they did not know algebra or trigonometry. High school mathematics up to and including trigonometry are also essential in map reading, communications, electricity and radio. They constitute the minimum mathematics required for courses in aviation, navigation, aeronautics, meteorology, ballistics, and engineering.

The practice of administrators of advising high school pupils against electing mathematics not only created a scarcity of men and women with a knowledge of mathematics but the teachers to take over the classes organized to make up this shortage were not available. Colleges and high schools now had to appeal to all teachers who ever had any experience with teaching mathematics to help out in this emergency. Many retired teachers were called back to active work. It is needless to say that under these conditions much time was lost and student effort wasted due to ineffective teaching. Even with good teaching mastery of mathematics was difficult to attain in the brief time allotment. The war has presented a forceful argument for the requirement of some mathematics of all high school pupils in the post-war curriculum.

War Discloses Needed Improvements in Teaching Mathematics

It must be gratifying to the teachers of mathematics that war has demonstrated the great importance of their subject to the successful conduct of the war and of the great industries. However, this also has brought to the surface some serious weaknesses in the teaching of mathematics. Many men and women of high general and mechanical ability found their progress retarded or limited because they evidently had received poor mathematical training. This has given rise to criticisms which cannot be waived aside. For they call attention to matters which have interfered with the war effort.

One interesting fact about these criticisms is that they are not new to the teach-

ers. For years they have been discussed widely in the mathematical literature. They call for reforms in teaching that should have been made long ago. The following are typical of the criticisms.

1. High school graduates are weak in the fundamentals of arithmetic. A large percentage of the students in training courses has not mastered the important facts and principles of arithmetic, all of which are supposed to have received much emphasis in the elementary school.

2. The students' preparation in the basic concepts and processes of algebra and geometry is poor. As in arithmetic the basic ideas are not understood, and therefore not retained, although they are in almost constant use in courses in mathematics.

3. Because the underlying principles are not understood, the tendency prevails to perform algebraic processes mechanically. They do not seem to have been learned but only memorized. Students do not remember them.

4. Schools do not develop the ability of clear and correct thinking in mathematical situations. The students have not the power to think through mathematical situations to form correct judgments, to make clear decisions and to draw logical inferences.

5. Students are inaccurate and lacking in thoroughness. They do not have confidence in their results. When speed is required accuracy decreases.

6. The students are not able to use the mathematics they have learned. They do not know how to apply their mathematical knowledge when they encounter unfamiliar situations.

7. Problem solving ability is low. Students lack the reasoning power to sove simple quantitative problems.

The Present Interest in Mathematics Should Not Be Allowed to Fade

It has been shown that in time of war mathematics takes a position of importance. The sudden demand for men and women with mathematical background creates a shortage of students and teachers which slows up the war effort. This can be avoided if the high schools in the future require some mathematical training of all pupils.

When the present war is ended there will be a letdown in military needs, and in military and technical training. However, the demand for men and women with such training will be greater than before the war. People in general will retain for some time the newly acquired interest in mathematics. It will be easier to attract pupils to the subject, especially if by improving instruction, criticisms like the foregoing can be avoided.

Proficiency in Arithmetic Should Be a Requirement for Graduation From the High School

Numerous studies have shown justification for the persistent criticisms that a large percentage of high school graduates is poorly grounded in arithmetic. To the complaints of the industries, business men, teachers of various school subjects and teachers of the later courses in mathematics, we may now add those of the Army and Navy. With them the arithmetical deficiencies raised a serious problem. "To enroll the necessary men" says Admiral Nimitz, "at training stations it sometimes was necessary to lower the standards in fifty per cent of the admissions. The lowest category of achievement as shown by tests was often in arithmetic."[2]

Traditionally systematic instruction in arithmetic is discontinued after the eighth grade. Courses in arithmetic are not, and should not be offered in the high school. Pupils are bored if they are required to start high school mathematics with a review of arithmetic, and a refresher course in the senior year just before graduation is

[2] Nimitz, C. W., "A Letter on the Need of Mathematical Competence in the Navy," The Mathematics Teacher, XXXIV (February, 1942), 88–89.

usually not more than a cramming procedure. Neither course promises lasting results or a solution of the problem. In the high school attention to arithmetic must be given in mathematics, in science, or in any other course whenever difficulties arise. Thus, the responsibility for arithmetical proficiency does not rest only with the mathematics department. Rather it is one of all teachers, *i.e.*, of the entire school. "Accuracy and facility," says the National Committee of 1923, "are of such vital importance to every individual that effective drill in this subject should be continued in connection with the numerical problems arising in other work."[3] Of course, the mathematics department has an excellent opportunity to contribute to this problem. However, alone it cannot accomplish what is needed, since only a comparatively small portion of the school population is enrolled in mathematics classes. Arithmetical proficiency should become a graduate requirement of the school. Every teacher then must be a teacher of arithmetic. Wherever in any class an arithmetical deficiency appears the teacher's responsibility is to do some corrective teaching, even if it means taking time out from his own subject. This is a very necessary phase of the plan for solving the problem, but it is not sufficient.

The school must plan a program which reaches all pupils and measures their arithmetical growth year by year over the entire school period. The first step is to administer a test on arithmetic essentials. The test should be simple and should not go too much into details. It should comprise the fundamental operations with whole numbers and common and decimal fractions, percentage, and problem solving. It should be such that it should not require more than about thirty minutes to take the test.

On the basis of the test results the pupils can be identified who are in need of further instruction in arithmetic.

The second step in the plan is to arrange for corrective teaching. This is a simple matter for the pupils who are taking courses in mathematics. It has been shown that the pupils taking three years of mathematics, tend to attain arithmetical maturity during the junior year without any special assistance. Many, of course, reach it earlier and some attain it in the senior year. Marked improvement always takes place in the junior year because of the computational work in the study of logarithms, and the large amount of arithmetic related to the work in science which most of the mathematics pupils take. With the proposed testing program this situation can be further improved. The teacher informs himself as to the processes in which the class is weak. He also identifies those individuals who are generally deficient. Accordingly, when in his class work an arithmetical deficiency becomes apparent he may reteach the entire class, or it may be sufficient to arrange for corrective work only with some of the pupils. Thus, he can remove all deficiencies before the course is finished.

Since the majority of pupils is not taking courses in mathematics special classes have to be organized for those who are weak in arithmetic. Usually one class meeting a week is sufficient. The work should not be merely drill, although some drill will be necessary. The core of the course should consist of problems taken from the quantitative applications in other subjects. Learning should be largely a task of assimilation, the emphasis being on understandings of such concepts as common and decimal fractions and of the principles on which the arithmetical processes are based.

The Teaching of All High School Mathematics Should Stress Understanding of Concepts and Principles

One of the major causes of retarding the progress of students in the special training

[3] Report of the National Committee on Mathematical Requirements. "The Reorganization of Mathematics," 1923, p. 6.

program was that they had failed to acquire an understanding of mathematical concepts and principles. The tendency among the students is to work mechanically by rules. This experience conforms to that of teachers of the upper mathematics courses in high schools and colleges. The following examples are typical of results presented by students who lack the necessary understandings:

$$\frac{3}{x}+\frac{4}{x}=\frac{7}{2x}; \qquad \frac{5a^2-2a}{3a}=\frac{5a^2-2}{3};$$

$$(a^2)^3=a^5; \qquad \frac{a^{-2}+b^{-1}}{a^{-4}+b^{-3}}=\frac{a^4+b^3}{a^2+b}.$$

The confusion in which the students who make such mistakes find themselves can be cleared away as soon as they understand the meanings of fractions and exponents, and the principles underlying the reduction of fractions.

Understandings develop slowly. Abrupt statements of definitions seldom carry with them any meanings. The practice is conducive to rote memorization and juggling of empty symbols. Meanings must grow out of repeated concrete experiences provided in the teaching. Definitions and processes which are outcomes of such experiences will be understood and the chances are that they will be retained. The following illustrations explain ways of developing meanings.

1. The literal number. "Letters in algebra are number symbols." Thus may the teacher of introductory algebra dispose of the concept as a simple matter in one sentence, but years later it is most disconcerting to the teachers of college algebra when some students try to distinguish between the number 5 and the letter x, or confess that they do not understand how $+a$ could be negative and $-a$ could be positive. There is not a page in an algebra textbook where the literal number does not appear. If its meaning is not understood these pages have little or no meaning to the learner. Manipulation of literal numbers becomes mechanical and makes little sense. Nothing is really clear. To attain the understanding the learner has to be led repeatedly to recognize that the letter is a number symbol, through a variety of situations. It may be positive, negative, specific, unknown, integral, fractional, irrational as in $\sqrt{2}x=5$ or $2\pi r=15$, general as in $ax=b$, an equation which represents an infinite number of equations, or a variable as in the function $3x+5$.

2. The exponent. The learner acquires this important concept slowly which to the teacher suggests serious teaching difficulties. Considerable time should be spent on developing "meaning" first before operations with powers are taken up. In the beginning the concept may be represented correctly through the study of areas of squares and circles. An abundance of practice in the problems relating to areas and using powers of the second degree will be helpful. Similar work then follows with volume formulas of cubes, cylinders, and spheres. The third step is to pass from squares, and cubes to powers of higher degree than the third. Finally, this leads to exercises involving a mixture of powers from the first to the fourth, fifth, etc., to the nth power. In all this work so far the exponent stands for a positive whole number. The expansion of the concept to include negative, zero, and fractional exponents should be deferred to a later time.

3. The algebraic processes. Many administrators and teachers believe that algebraic processes are largely a matter of rules developed to the point of automatic response by drill. If this is called "teaching algebra" then the supposedly great advantage of the method is that "anybody can teach algebra."

Moreover, many pupils take kindly to a method of work that is purely mechanical. They have used it with success in arithmetic. If they can get by with it in algebra and make good grades, their faith in it is greatly strengthened. However, they are in for a shock when they study geometry. For geometry is not a subject to be mas-

tered by mechanical repetition, but a way of thinking. They try to memorize the proofs of theorems and exercises and to reproduce them from memory, but soon discover that they are unable to retain such a large amount of subject matter. Until they realize this it is almost impossible to induce them to change their method of study. Often it is too late to repair the harm that has been done.

To be sure, there must be "rules" and "cases," but the method of deriving them is very important. They must grow slowly out of experiences which require mental effort, careful thinking, and time. The good teacher knows that too-early mechanizing of a process is fatal. Step by step he leads his pupils slowly to the rule. First he makes use of suitable exercises intended to aid the learner to discover the rule independently. It is then correctly formulated by the teacher. Next it is applied to specific problems, but for some time it is checked by reasoning until it is clearly understood. Then drill follows to enable the pupil to respond quickly and accurately the way it is done by the mathematician. This slow procedure requires care and patience, but it pays dividends later. The following illustrates the method with the pupils' first experience with one of the laws of exponents.

Exponents at this stage are positive whole numbers. First the pupil multiplies powers by counting the number of times a factor enters in the product. After he has done this for a certain time he discovers that the result may be attained quickly by adding the exponents. The number of examples necessary to make this adaptation varies with the individuals and the teacher must be careful not to cut short these experiences. The correct statement of the law is now formulated by the teacher, in particular it is emphasized that it applies only to powers having the same base. The law is then applied to new problems, each result being verified by counting equal factors. Finally, the law is used with only an occasional verification when the teacher is in doubt as to the pupil's understanding. This is the drill period for the purpose of making responses automatic.

The foregoing argument was directed against "too-early" mechanization of processes that ultimately must become mechanical. However, some processes in algebra that are purely mechanical are quite unnecessary. Since they give rise to so much confusion and so many errors they might as well be discarded. One typical example is the process known as "removal of parentheses." It really is not a process at all. Rather it is the end to be attained by means of some fundamental process.

Before the pupil reaches the place in the course where the process is presented he usually has learned to add, subtract and multiply monomials and polynomials. Now the polynomials come enclosed in parentheses preceded by a plus sign, a minus sign, or a number symbol. The problem is to "remove" the parentheses. What the pupil actually has to do is to carry out the indicated addition, subtraction, or multiplication. When this has been done the parentheses are no longer needed and are removed. A special mechanical rule for "removal of parentheses" is therefore quite unnecessary.

Another example of a mechanical process which is really not needed is "cancelling." In general the process refers to reduction of fractions but it is also used in simplifying polynomials. Mechanical cancellation without a knowledge of the actual principles involved leads to no end of confusion and difficulties. However, it is so well established as a process in arithmetic that it is hard to drop it in algebra. The pupil who finds correctly by cancellation that

$$\frac{\overset{2}{\cancel{14}x}}{\underset{5}{\cancel{5}}} \cdot \frac{\overset{5}{\cancel{25}y}}{7} = 2x \cdot 5y$$

might fail to see why he should not obtain from

$$\frac{14x}{5} = \frac{25y}{7}$$

the result $2x=5y$ by cancellation.

A third example of an unnecessary mechanical process is "transposition" used in equations. The pupil is taught to solve $x+3=18$ by transposing the 3 and changing its sign. In a case like $x-3=18$ the process may be of help, but he should solve $x+3=18$ by subtracting 3 from both sides, i.e., the addition in $x+3$ is undone by using the principle that equal numbers subtracted from equal numbers leave equal remainders. Once this idea is grasped all future equations are solved similarly: The pupil identifies the process to be undone and then employs the inverse process. Thus the thinking involved in $x+3=18$, $x-3=18$, $3x=18$, $x/3=18$, $\sqrt{x}=5$, $x^2=25$ is really the same, and a special process of transposition for the first two is not necessary.

The pupil whose training in algebra consists largely of such mechanical stunts as removing, cancelling, transposing, clearing of fractions and cross multiplying does not learn algebra as a way of thinking, nor is he grounded in the underlying principles. The result is disheartening to the students who in later courses try to understand what they are doing. The following are a few samples of board work done by students in a specialized training course:

$$\frac{3}{7}+\frac{4}{5}=\frac{3+4}{7+5},$$

an imitation of

$$\frac{3}{7}\times\frac{4}{5}=\frac{3\times4}{7\times5}$$

$\sqrt{x^2+y^2}=x+y$, suggested by $\sqrt{x^2y^2}=xy$.

$48x+35y$, obtained from

$$\frac{24x}{5}+\frac{7y}{2}$$

and apparently suggested by the cross multiplication used in solving the equation

$$\frac{24x}{5}=\frac{7y}{2}.$$

Facility in Using Mathematics in New Situations Should be a Major Objective of Teaching

A pupil's knowledge of mathematics is of little value if he is not able to use it in new situations. Moreover, ability to use mathematics is a real test of his mathematical powers. Thus, if he can solve $3r=36$ but not $2\pi r=36$ he has not yet acquired the full meaning of a simple linear equation in one unknown. If he can solve $6x^2-5x+1=0$ and recognizes that $6\sin^2 x-5\sin x+1=0$ is of the same form and therefore may be solved by the same method, he has developed ability to use his knowledge of mathematics. A pupil who knows algebra should prefer the algebraic method of solving problems to the more tedious arithmetic method. He should have no difficulty in physics to use his knowledge of solving $a=b/x$ for x to solve $I=E/R$ for R. He should be quick in recognizing that by solving the system

$$I_1R_1+(I_1+I_2)R=E$$
$$I_1R_1-I_1R_2=0$$

where R_1, R_2 and R have given numerical values he can find the electrical currents I_1 and I_2 by the same method which he uses to solve

$$3x+2y=6$$
$$5x-8y=0$$

for x and y.

The criticism made by teachers of military courses that the students cannot apply the mathematics they have learned to military problems suggests that still more stress should be laid on this type of training. In addition to the customary applications taken from business, industry, and school subjects other than mathematics, an abundance of material is now available in textbooks on aviation, navigation, electricity, and radio. Here we find formulas like the following.

Lifting ability of a plane:

$$L=C\frac{p}{2}SV^2.$$

Formula for drift:

$$D=\frac{akbc^2D'}{n\cos^3\phi}.$$

Horizontal range of a projectile:

$$R=v_0^2\frac{\sin 2\theta}{g}.$$

Path of a projectile:

$$y=x\tan\theta-\frac{gx^2}{2v_0^2\cos^2\theta}.$$

Centrifugal force:

$$F=\frac{W}{g}\times\frac{4\pi^2r^2cd}{60^2}.$$

An effective procedure of teaching the use of mathematics has been illustrated in twenty attractive posters designed by "Signal Schools of the Sixth Army Command, 20 N. Wacker Drive, Chicago, in Collaboration with the Men's and Women's Mathematics Clubs of Chicago." The illustrations require a knowledge of geometry to understand the diagrams and to interpret graphical representations of formulas. They exhibit the evaluation of formulas used in radio, such as

$$X_L=2\pi fL,\ A=\frac{\mu R_L}{r_p+R_L},\ |Z|=\sqrt{R^2+X_L^2},$$

and

$$f=\frac{1}{2\pi\sqrt{LC}}.$$

Solutions of equations are shown. Thus,

$$L=\frac{N^2R^2}{9R+10\dfrac{N}{r}}$$

is transformed to

$$R^2N^2-\frac{10L}{n}N-9RL=0$$

which is then solved by the formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Solutions and evaluations of formulas taken from radio are preceded by solutions of corresponding formulas familiar to the student of algebra. For example, the solution of

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

for C is illustrated by solving

$$\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

for x; the solution of

$$\frac{x}{2L-x}=\frac{R_1}{R_2}$$

is preceded by that of

$$\frac{x}{a-x}=\frac{b}{c};$$

and the computation of K from

$$K=\frac{\sqrt{c_1c_2}}{\sqrt{c_m+c_1}\sqrt{c_m+c_2}}$$

is shown to be like that of x from

$$x=\frac{\sqrt{ab}}{\sqrt{c+a}\sqrt{c+b}}.$$

Military Problems May Be Used to Contribute to the Development of Problem-Solving Ability

The complaint that students in military courses show a lack of problem-solving ability is not surprising. Teachers of mathematics in the high schools and colleges find the development of this ability a slow and tedious process. High school pupils have always regarded the "problems" as the most difficult part of mathematics. Problem-solving cannot be mastered by drill on rules for specific types of problems. It slowly grows out of practice with ever so many problems involving the largest possible variety of types. The *Committee of the Progressive Education Association* places so much importance on the development of this ability that it devotes to it

seven chapters of its yearbook, each chapter being concerned with one ability in problem-solving. For "The development of intelligence in analyzing problem situations, otherwise referred to as reflective thinking, although but a part of the purpose of education, is so essential as to be given a major place."[4] Likewise the *Joint Commission* assigns a prominent place to "applying the technique of algebra in problem situations arising in business in the shop, in science and in everyday life."[5] Some writers have recommended that the study of problem situations be the basis of the entire mathematics curriculum.

Unsatisfactory problem material has been blamed partly for the failure to develop problem-solving ability. If a problem is unreal or falls outside of the pupil's experience its training value is necessarily small. Real problems, however, are hard to find. Hence, military problems may in the future replace much of the artificial problem material of the textbook. These problems are real, interesting to the pupil and can be understood by him. A knowledge of the usual mathematics offered in high school algebra and geometry is sufficient for the solutions of problems involving rate, time and distance of planes, destroyers and submarines; centrifugal force of a bomber; relation of lift and velocity; relation of weight and distance from the center of the earth; and relation of lift and wing area.

Influence of the War on the Mathematics Curriculum

Mathematics has always been of importance to those high school pupils who plan to continue their education in college, especially the future scientists and engineers. In the reports of the two national committees which were published in 1940 ample evidence was presented to show that mathematics also has an important place in general education. The present war has left no doubt that mathematics is essential to the armed forces and industries. When the war ends some military training will be continued. Moreover, the industries will carry on a large amount of scientific investigation. Thus, after the war the tendency will be toward a greater peace-time demand for men and women with mathematical training than ever before. The high schools should prepare to meet these needs by attracting a larger number of pupils to the study of mathematics. What should be the content of the mathematical curriculum?

The recommendations made by the *Joint Commission* and the *Committee of the Progressive Education Committee* have suggested improvements in content and methods which should be put into practice by the teachers of mathematics. To these recommendations will be added the advice of new committees, such as the recent *Committee on Post-War Plans.*

The military authorities have suggested no radical changes in the mathematical curriculum. In fact, they have expressed the opinion that a course in mathematics comprising arithmetic, algebra, geometry, and trigonometry are entirely satisfactory for their needs, with the recommendation that arithmetical proficiency should be made an aim of secondary school mathematics. New courses are not recommended or even desirable from the point of view of the Army and Navy. Courses, such as recommended by the *Joint Commission* should meet the needs of military, industrial and college groups.

However, the military authorities have been very emphatic that improvements in methods of instruction are imperative. Present methods fail to train a large percentage of pupils in accuracy and thoroughness, in clear thinking, in acquiring understandings, and in ability to use and

[4] Committee of the Function of Mathematics in Education. Progressive Education Association. *Mathematics in General Education*, New York. D. Appleton & Company, 1940.

[5] *The Place of Mathematics in Secondary Education.* Fifteenth Yearbook of the National Council of Teachers of Mathematics, New York: Teachers College, Columbia University, 1940.

apply the mathematics they have learned. The war thus gives new impetus to the movements of developing a mastery technique of teaching; of offering wide experiences to secure transfer to new situations; and of planning activities which give training in clear thinking.

The War Has Emphasized the Need of Mathematical Instruction in General Education

In an emergency of war the armed forces are drawn mainly from the large group of men and women who have had a general education but no special training in mathematics. Recently the importance of mathematics to this group has been more and more recognized. The needs of the armed forces emphasize this still further; for the same type of mathematics serves both purposes.

General education aims to prepare the pupils for efficient participation in the activities in which they engage later as adults and for the assumption of their share of social responsibility. Mathematics has much to contribute to these aims. It is useful in the life of the people at home, in their daily work, in business, in industry, or wherever people must be competent to deal with quantitative problems. The mathematics of the home includes arithmetic in dealing with bills, accounts, making purchases, earning a living, budgeting an income, saving and borrowing, investments, insurance taxes, etc. Geometry is also useful in the daily life of the people. It acquaints them with the geometric forms in nature, art, and architecture, trains in space perception and space imagination and develops skill in the use of the instruments of geometry. Moreover, everybody who wishes to be an intelligent reader of the current literature should be acquainted with algebraic symbolism, be able to interpret graphs, and have a fair understanding of simple formulas.

Boys and girls expecting to become skilled workers in the shop and industries of the community need shop mathematics in which arithmetical proficiency is essential and which includes such geometry as measurement of lines, angles, surfaces and contents; knowledge of precision and accuracy; the use of verniers, slide rule, tables and logarithms; and ability to interpret scale drawings, blueprints and graphs. Algebra also is needed. The basic concepts must be understood and workers must be able to evaluate formulas and solve simple equations.

Mathematics aids further in general education because other school subjects increasingly make use of the subject. Quantitative problems arise frequently in the arts and sciences. A knowledge of algebra, geometry and often numerical trigonometry is required in the solutions of these problems. Thus every high school graduate should acquire arithmetical skills and a knowledge of basic algebra and informal geometry.

Mathematics has a further share in the accomplishments of the purposes of general education because it offers opportunities for the development of efficient study habits, intellectual independence, and clear thinking in forming correct judgments, making decisions, drawing logical conclusions and evaluating situations.

A study of various reports on pre-induction mathematics discloses the fact that the mathematics suitable for general education do not differ from the minimum required for the armed forces.[6] Pre-induction outlines stress the importance of the development of such habits as thoroughness, accuracy and clear thinking. The instructional materials are the essentials of arithmetic, informal geometry, algebra and numerical trigonometry. The war should have considerable influence to strengthen the demand for a course in a type of socialized mathematics to be required of all high school pupils who are not taking the traditional mathematics course offered by the school.

[6] For details see the "Report of the Committee on Pre-induction Courses in Mathematics," The Mathematics Teacher XXXVI (March, 1943), 114–124.

The Hyperbolic Analogues of Three Theorems on the Circle

By JOSEPH B. REYNOLDS
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THE THREE THEOREMS of plane geometry that follow are closely related.

A. If through a fixed point within a circle any chord is drawn, the product of its two segments has the same value, in whatever direction the chord is drawn.

B. If through a fixed point without a circle a secant is drawn, the product of the whole secant and its external segment has the same value, in whatever direction the secant is drawn.

C. If through a fixed point without a circle a tangent to the circle is drawn, and also any secant, the tangent is a mean proportional between the whole secant and its external segment.

In fact, these three theorems are particular cases of the more general theorem following.

D. If one of two lines intersecting in O cuts a circle at A and B while the other cuts it in A′ and B′ the product (OA)(OB) of the segments of the former equals the product (OA′)(OB′) of the segments of the latter.

If OA is a directed line the product $(OA)(OB)$ is positive when A and B are on the same side of O and negative when they are on opposite sides of O. Hence, for directed segments, the ratio $(OA)(OB)/(OA')(OB')$ may have the value $+1$ or -1 and still leave the products of the undirected line segments equal. If O lies inside $x^2+y^2=a^2$ each product is negative, if outside each product is positive; so for the circle the ratio is always $+1$. It is the purpose of this paper to inquire into the conditions necessary to make the ratio $+1$ or -1 when the points A, B, A', B' lie on one or other of the conjugate equilateral hyperbolas whose equations are $x^2-y^2=\pm a^2$. Certain interesting corollaries will come out of the inquiry.

Let the equations $x^2-y^2=\pm a^2$ of the hyperbolas be transformed to refer to the intersecting lines as axes. Let the new origin O have the coordinates (h, k) and let OA and OA' be inclined to the X-axis at the angles θ and ϕ, respectively. The equations of the hyperbolas, referred to OA and OA' as X-axis and Y-axis respectively, become

$$(h+x\cos\theta+y\cos\phi)^2 - (k+x\sin\theta+y\sin\phi)^2 \pm a^2 = 0$$

or

$$x^2\cos 2\theta+2xy\cos(\theta+\phi)+y^2\cos 2\phi+\cdots +h^2-k^2\pm a^2=0.$$

Since OA and OB are the roots of this quadratic equation, when y is set equal to zero, we see that

$$(OA)(OB)=(h^2-k^2\pm a^2)/\cos 2\theta$$

and in like manner

$$(OA')(OB')=(h^2-k^2\pm a^2)/\cos 2\phi.$$

Letting the ratio $(OA)(OB)/(OA')(OB')$ be designated by R, we have

$$R=(h^2-k^2\pm a^2)\cos 2\phi/(h^2-k^2\pm a^2)\cos 2\theta = rr',$$

in which

$$r=(h^2-k^2\pm a^2)/(h^2-k^2\pm a^2) \quad \text{and}$$

$$r'=\cos 2\phi/\cos 2\theta.$$

We note the following cases.

1. When the points A and A' lie on the same curve the signs before a^2 in r are the same and $r=1$.
2. If O lies on an asymptote so that $h^2=k^2$ and the point A lies on one curve and A' on its conjugate $r=-1$.
3. If $\theta=-\phi$, so that the lines are equally inclined to the axis of either curve, $r'=1$.

4. If $\theta = 90° - \phi$ the lines are mutually perpendicular and $r' = -1$.

A combination of either one of the first two of these with either one of the second two makes $R = \pm 1$ and so meets the requirement sought.

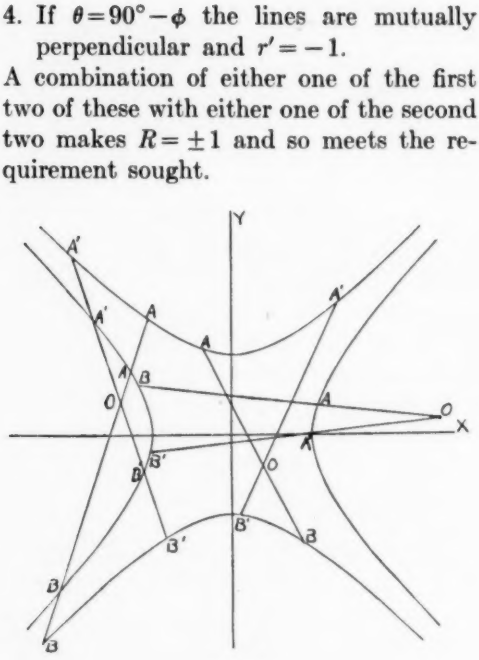

FIG. 1

The different situations can be covered by the following theorems.

Th. I. If one of two lines intersecting in O cuts an equilateral hyperbola in A and B and the other cuts it in A' and B' the products of the segments $(OA)(OB)$ and $(OA')(OB')$ are equal provided the lines are equally inclined to the axis of the curve or are mutually perpendicular.

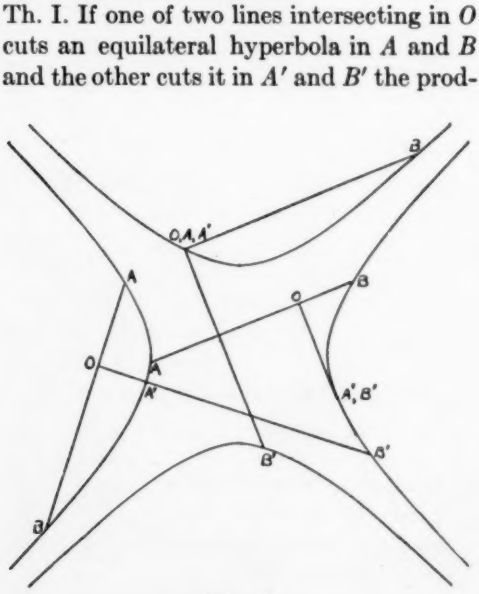

FIG. 2

Figure 1 shows examples of different positions of lines equally inclined to the axes of the curves. For the situations pictured here $R = 1$ since, in every case the products of the segments $(OA)(OB)$ and $(OA')(OB')$ have the same sign.

Figure 2 shows examples of different positions of the lines when they are mutually perpendicular. One general and two particular positions are shown; the latter being where the point O lies on the curve and when one line is tangent to the curve. When O lies on the curve the product of the segment is zero. In other positions it is seen that for one line A and B lie on opposite sides of O while for the other line A' and B' lie on the same side of O making R have the value -1. The position in which OA' is tangent to the curve making A' and B' coincide leads to the following corollary.

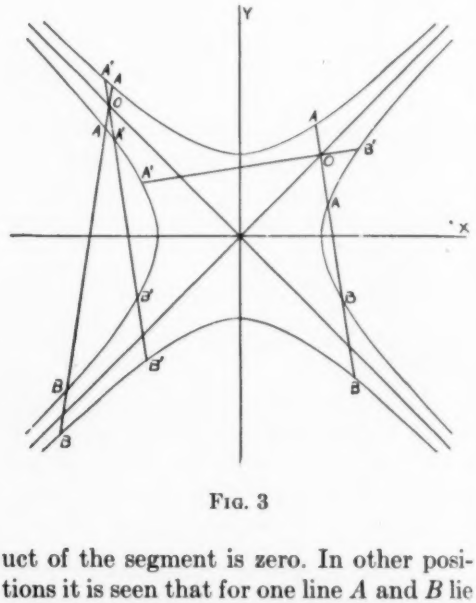

FIG. 3

Cor. 1. The length of the tangent from any point O to the equilateral hyperbola is a mean proportional between the segments into which O divided the chord of the hyperbola through O perpendicular to the tangent.

Th. II. If one of two lines intersecting in O, a point on an asymptote of a pair of conjugate equilateral hyperbolas, cuts one hyperbola in A and B and the other cuts

either hyperbola in A' and B', the products $(OA)(OB)$ and $(OA')(OB')$ are equal provided the lines are equally inclined to the axes of the curves or are mutually perpendicular.

This theorem is true since it takes in all the situations covered by the four cases above when O is on an asymptote and no others. Figure 3 shows examples of different positions of the lines and points such that $(OA)(OB)=(OA')(OB')$ in which A and B are points on the same curve and likewise A' and B' are points on the same curve, both branches of each curve being considered part of that curve. A special case of this theorem gives rise to the following corollary.

Cor. 2. If two parallel chords of an equilateral hyperbola intersect a tangent to the hyperbola at points on the asymptotes, the chords are diameters of circles intersecting at right angles at the point of tangency and the line joining the center of the chords is a diameter of the circle having a common tangent with the hyperbola.

In Figure 4 let OP be the tangent drawn at P cutting the asymptote in O and OA and OB the segments of the chord perpendicular to OP; then $(OA)(OB)=(OP)^2$. Thus, P is a point on the circle having AB as diameter. If the tangent at P cuts the other asymptote in O_1, like reasoning shows that the chord A_1B_1 drawn perpendicular to O_1P at O_1 is also the diameter of a circle passing through P. These diameters are parallel chords so the line CC_1 joining their centers is a diameter of the hyperbola conjugate to one parallel to AB.

Since conjugate diameters of an equilateral hyperbola make equal angles with the asymptote on opposte sides, the angles CEO and COE are equal (E, the origin).

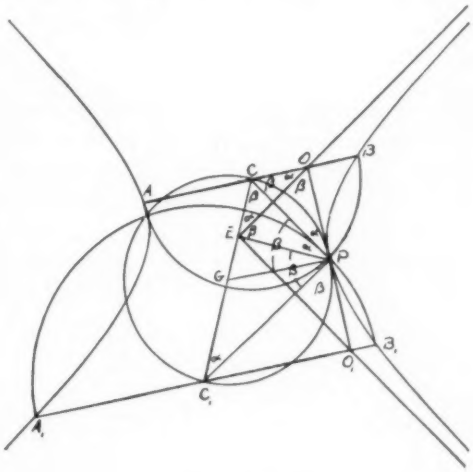

FIG. 4

Since P bisects OO_1, $PE=OP$ and angle $PEO=$angle POE and $CEP=COP$ is a right angle. Hence, angle $CPE=$angle COE and angle $EPC_1=$angle EOP. That is, the two circles intersect at P at right angles. The other point at which they intersect (at right angles) is also on the hyperbola and determines the tangent parallel to OP.

If G is the center of the circle described upon CC_1 as diameter the triangles GPC and GPC_1 are isosceles so that the angle $GPC=$the angle $PCG=$the angle PCO. GP is, therefore, parallel to CO and perpendicular to OP at P, proving that the circle with center at G and the hyperbola have a common tangent at P.

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The Latent Proportional Parts Table in Ordinary Interpolation

By Wm. Fitch Cheney, Jr.
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Ordinary interpolation is probably used more widely than any other process in arithmetic, save only the fundamental ones of addition, subtraction, multiplication and division. The object of this paper is to point out the presence, in every logarithm table, of a hidden subsidiary table of proportional parts. This may be called the Latent Table of Proportional Parts. Its use is recommended with all logarithm tables not equipped with auxiliary tables of Proportional Parts, as it saves time without producing unusual errors.

There are two principal reasons why ordinary interpolation is not absolutely accurate. In the first place, the graph of the function represented by the table in which we wish to interpolate, is necessarily a curve. Ordinary interpolation, however, locates points, not on the arcs of this curve, but on its chords. The second source of error in interpolation is the process known as rounding off. If we have two random numbers each rounded off to the nearest integer, and if we add or subtract them, we will find that once in four times we get a different result from what we would have gotten, if we had added or subtracted the numbers first, and rounded them off afterward. The error introduced by rounding will ordinarily be anywhere between plus and minus half a unit in each number. Half the time, these errors due to rounding in our two numbers will be in offsetting directions, and hence our results must be the same, whether we rounded off first or last. When the errors due to rounding off two numbers lie in the same direction, their sum is equally likely to be anything from zero to one. Hence in twenty-five per cent of all cases the combined error will exceed one half. This introduces an error of one between rounding first and then adding, as against adding first and then rounding. This source of error is inherent in ordinary interpolation. Most of us have either been unaware of it or have ignored it.

In order to be specific, we will discuss interpolation in an ordinary five-place table of logarithms. If no Proportional Parts table is provided, the usual process of finding the logarithm of a five-place number is to use its first three digits as a line indicator, its fourth as a column indicator, and its fifth digit as a multiplier in the interpolation. The interpolation is performed by subtracting the logarithm listed for the first four figures of the given number, from the next listed logarithm. This is called the Tabular Difference. This Tabular Difference is then multiplied by the fifth digit of the given number, preceded by a decimal point, and the product is rounded off to produce our Correction or Problem Difference. This Correction is then added to the logarithm of the four-place number already found, to obtain the desired logarithm of the given five-place number. When Proportional Parts Tables are provided, they ordinarily perform for us only the multiplication in this process. In some tables however, the Proportional Parts figures appear at the right of each line of the table, giving the Corrections already rounded off for the various possible fifth digits for use in that line. This latter, less common type of Proportional Parts table ignores the fact that the tabular differences within one line of a logarithm table may themselves differ. This is not a serious error, as it should rarely (if ever) produce a difference of more than one in the last figure of our logarithm; and as we have already seen, interpolation with or without the commoner type of Propor-

tional Parts Tables frequently does the same thing.

It is rather surprising to most people to find that a logarithm table in which no Proportional Parts Tables are openly printed, does nevertheless contain a Latent Table of Proportional Parts. This is due to the steady increase in the logarithms of numbers. At the beginning of a five-place table, the difference between successive logarithms is approximately .00043. This, of course, is the product of the Modulus M (i.e. $\log_{10} e$) by the fraction dx/x, where x is the number whose logarithm is sought and dx is the constant interval between the numbers listed in the margins of the table. At the end of the table this difference between successive logarithms has shrunk to about .00004. The logarithm appearing in the zero column of any line will differ from the logarithm listed in column n of that line by approximately n times the average tabular difference in that line. Consequently, n-tenths of the average tabular difference in any line may be closely approximated by subtracting the logarithm in column zero of that line from the logarithm of column n in that line, dividing by ten and rounding off one figure.

The reader is urged to refer to a table of five-place logarithms while reading the examples which follow. Let us now, for instance, find the logarithm of 4.7283. We look on line 472 and in column 8 of our table and locate the logarithm, .67468. We then subtract the figure .67394 found in column zero from the figure .67422 listed in column 3. This difference of .00028, divided by ten and rounded off one place gives .00003. This .00003, added to the .67468 gives .67471. This is the desired logarithm of 4.7283.

For interpolating backward from a logarithm, L, to the corresponding number, we subtract the largest logarithm in the table, which does not exceed L, from L. We multiply this difference by ten and add it to the logarithm in column zero on the same line of the table. We then look for this result in that line, and use as the fifth figure of our number the heading of the column containing the logarithm nearest this result. For example, to find the number whose logarithm is .43213, we find the logarithm .43201 on line 270, column 4. The difference between these two logarithms is .00012. Ten times .00012 is .00120. We then add .00120 to .43136, which appears in column zero of that same line, getting .43256. This is closer to .43249 than to any other logarithm on that line. The heading of the column showing .43249 is seven. Consequently the number corresponding to our original logarithm, .43213, is 2.7047.

This use of the Latent Proportional Parts table present in any five-place table of logarithms, gives approximately the same accuracy as the commoner methods of ordinary interpolation. It introduces no appreciable error about three-quarters of the time, and an error of one in the last decimal place about one-quarter of the time. But so does ordinary interpolation, as we have seen.

It avoids the need of the usual, auxiliary Proportional Parts Table. It still performs the relatively difficult multiplication mechanically. It may, of course, be readily adapted for use in four or six place tables of logarithms, or for use in any other tables in which the numbers are arranged in ten columns headed from zero through nine. The author has found this process quite teachable and would be interested to hear from others who may see fit to experiment with it and then send him their ideas about its usefulness.

Let Us Be Sensible About It

By IRWIN A. BUELL
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BOOKS telling us of education in general and those dealing with the teaching of mathematics in particular; lecturers who try for congenial responses from their audiences; psychologists both pseudo and honest; scholars trained narrowly but speaking broadly; political educators and educational politicians; *et cetera; et cetera* have all said it and have emphasized it and resaid it; and still it may not be true. One still does not have to know the why of *everything* in order to enjoy life and to add to human happiness, and this is as true in mathematics as it is in other fields.

One can use the telephone without knowing the basic theory of its operation. Or one can know the basic theory and yet not have it in mind every time one answers its ring. Most people will never know the theory; nor does that matter much. Such illustrations could be multiplied endlessly. Then why, Oh why, must every child be taught so many underlying reasons in the field of mathematics. He does not need to know the scientific reason back of everything he does so long as that which is done is done correctly.

There is a whole theory of numbers that the child cannot understand at his level of maturity, yet this does not mean that he should not use numbers. This is, or at least should be, very platitudinous, but needs to be said because of the somewhat illogical frame of mind of many educators who accept sweeping statements, which sound just right, without giving them an analysis that would show that they are not. How many times has it been said, "Teach him to understand everything as he goes along," and how few have questioned the propriety of such a direction, even when an analysis of it would show the impossibility of it.

It is not necessary for the many to be steeped in theories of mathematics. They are beyond the ability of most. Their value is mainly technical. There is little of sociological or humanistic significance to be squeezed out of them. They are for the producer, not the consumer. And since they are beyond the comprehension of many, their disciplinary value for these is extremely low or nonexistent. But many do need to be able to use numbers and formulas as part of their daily routine. Let us not be disturbed in our own minds when we teach them only that which has value to them.

Article after article has been printed in which the author has said that the pupil in mathematics must consciously understand each time he performs a definite operation just the reason why he is permitted to do as he does. I should like to take exception to such statements. It slows things down too much and is quite unnecessary. There are things to be done, that have to be done, that can be done without all the theory. Furthermore since the mathematical ability of many is low and the total amount they will learn is limited, it is best to reduce many things to routine so that they may go on farther with practical work than they could go if we insisted on "completeness" all along the way. For people of ability it is also well to eventually reduce elementary things to routine to free the mind for more debatable philosophical questions at a higher level. This is true whether in mathematics or in every day life, and we should encourage such a procedure.

One difference between some professional mathematicians and myself is that they want to have a lot of "meanings" "taught into" mathematics at its lowest levels where it is doubtful much can be done that is worth while, whereas I would make these levels fairly automatic even for able people, and free their minds for a study of meanings at a higher level, as high a level as the students can reach, where

more can be done. Transfer of training, if any, is worth much more at the higher level.

As we go along with our present methods we advance farther and farther beyond the point where most pupils will have a great deal of practical use for the mathematics they learn. This might be used as an argument for more explanation of *basic* reasons. On the contrary, however, mathematical "reasons" are the things we have less use for and forget most easily. Applications are remembered because used. The change of sign in transferring terms is remembered, the basic reason is forgotten.

In the Sixteenth Year Book of the National Council of Teachers of Mathematics we read "The significant difference between the program of arithmetic which finds support in this year book and that of a decade ago is in the extent to which children see meaning in the numbers which they use and operate." Just what meaning do they now see in the number 4 that they did not see ten years ago. Are we going back to the mystic numbers of centuries ago? "The keynote of the new arithmetic is that it should be *meaningful* rather than *mechanical.*" I say lets make it increasingly mechanical and then go on to something more abstract. Let us continually make the difficult into the mechanical, and go on to the more difficult.

Let us not look for "meanings" that are not there. Let us not become too mystical about this whole business. And please, let us not load up our own minds and the children's minds with round-about philosophies, methods, procedure, "meanings," *et cetera, et cetera.* Let us teach him that 8 and 6 are fourteen; that is all there is to it. He'll learn that as a definite fact without a lot of lacework built around it; and it has all the meaning it needs to have, the right kind of meaning, the speedy, definite meaning.

One may say "Why that has no intellectual content." Well, d-o-g spells dog and one had better learn just that without looking for a lot of intellectual content in back of the fact because such has little, if any, value. The war should teach us a few things of that kind. It should teach us that the most valuable man in the world is one who can produce results of value to humanity with the smallest cost and least delay. It should also teach us that it is rather useless to train all sailors as admirals or all soldiers as generals. If one is to be either, he will get that training later. May we have less meticulousness and picayuneness but more of the larger view, a better perspective toward the goals of education in general.

Of course when the explanation is simple and easily given, then it should be taught to him and he should be expected to understand it. But even then, if he develops a direct and simple "short cut," or if one is taught to him, as it should be, if one exists, then his thoughts do not need to wind through the more devious corridors of the "logical" or "mathematical" derivation every time he needs to achieve a certain result.

As was pointed out, some explanations are beyond the comprehension of the child, and others are so weak that they had better not be attempted. I have neither seen nor heard an explanation, suitable for a ninth grade pupil, of the reason why the product of one negative number by another negative number is itself positive. I have listened to attempted explanations at first hand or have had them passed on by pupils. None was good and some were downright silly or absolutely untrue. Why attempt an explanation then?

Some other explanations can and should be given, but as soon as understood, should be ignored in all practical work. Transposing of terms in algebraical equations is a practical operation and will be done without thought of the underlying justification, even though such justification should be understood at some time. The formulas for differentiation in calculus are used without recalling each time that "Δx approaches zero" even though the long or general method was once taught them.

Is it better to spend a lot of time to clear

up the underlying logic of one point, as it were levelling off each hillock as we come to it, or better to outflank, by-pass, some of the more difficult questions and go on? If later on they happen to attack us from the rear we will turn and demolish them then when it is easier to do so. Most will never threaten seriously. Our passion for completeness should not be allowed to slow down our progress too much. Washington told Braddock that.

Permitting the reduction to the habit level, of certain processes also makes more natural the introduction of topics or rules the reasons for which are beyond the maturity, or degree of advancement, of the pupils at their various levels, but which are quite essential for their other work at that point. The pupils are quite willing to make use of such theories unless they have been so indoctrinated with the idea that they should not use anything they do not understand, that they are all confused at the idea and even consider the teacher inefficient and scatterbrained and perhaps a bit dishonest when he presents such a thought. Most children learn to tell time and use the electrolux without having to have all the mechanism of each explained to them. It is essential they should in this world of ours.

The following are a few illustrations to give more substance to this thesis:

I. A few relationships whose bases we should keep in mind continually in our mathematical work.

1. Completing the square when necessary, as in analytical geometry, when dealing with circles, etc. whose centers are not at the origin;

2. Problems involving percentages;

3. Inverse functions in trigonometry;

4. Problems in probability;

5. The securing of the median or the average deviation in statistics; *et cetera.*

II. A few relationships whose bases should be understood at some time but can eventually be ignored in favor of a formula or rule-of-thumb method in practical work:

1. Binomial theorem;

2. The formula for solving quadratic equations;

3. Most formulas for differentiation;

4. Inversion in division of fractions;

5. Transposition with change of sign;

6. Sine formula for solving triangles;

7. The so-called squared formulas in trigonometry;

8. The formula, or method, of determining the area between a curve and a coordinate axis; *et cetera.*

III. A few situations where a rule or formula or method should be used, at least at first, without proof or derivation.

1. Division or multiplication of negative numbers;

2. Prismoid formula for volumes;

3. Half-angle formulas in trigonometry;

4. Use of determinants in solving simultaneous linear equations;

5. Many formulas in integration;

6. Formula for radius of inscribed circle;

7. Formula for circumference and for area of a circle; *et cetera.*

These ideas are neither new nor radical. Most teachers of mathematics do follow some such outline. But many who do, do so apologetically and listen with superficial rapture to speakers, who tell them they must never ask the child to accept anything without mathematical proof, never take anything on faith. Why be shamefaced about a practice which experience has proved is the most workable and valuable and therefore the best one.

Do we really mean to say the student should not have faith in something he can use, something he knows is right, even though he does not know why it is? Teachers of mathematics do not really mean that even when they say they do. Why must we be illogical ourselves in order to give lip service to the logic of our subject?

This is not naivete, and it is hoped that it will not be considered treason. It is written in a spirit of constructiveness, with the hope that many will give it thought and, some at least, support. Its underlying motif is one of realistic idealism, one of facing things as they are, adjusting to them and going on from there.

Schools and Colleges Face a Challenge

By J. M. Bledsoe

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For the past several years there has been a steadily increasing shortage of adequately trained teachers of mathematics and the natural sciences in the public schools throughout the country. This has been true especially of mathematics.

It is unnecessary at this time to enumerate the various reasons responsible for and leading up to this situation. It is important, however, that attention be called to these facts, and some effective remedial steps be taken to relieve the condition as best we can. It should be remembered that the subject of mathematics has been almost completely eliminated from the high schools throughout the country, and that not a single hour of the subject is required for any college degree in the teacher-training institutions.

Shortage to Become More Acute

It is clearly obvious that a more serious shortage of qualified teachers of these subjects, so vital in our defense program, faces the public schools for the immediate future. The high schools have been dependent largely upon young men to teach mathematics, physics, chemistry, and the other sciences. Many of these young men soon are to be called into the defense service. Their places must be filled by young women, if filled at all. There is no doubt but that our young women are both capable and willing to meet this challenge in this hour of the nation's crisis.

The schools cannot afford to neglect this vital phase of training so essential to meet the crisis that now faces the country. We need not only trained teachers for the public schools, but those who enlist in the nation's defense service must have this training also.

Suggestive Remedy

In order to meet the needs of the immediate future, and to avoid a similar bottle neck in the future of a shortage of teachers of these vital subjects, the mathematics department of the East Texas State Teachers College submits suggestive plan for the preparation of teachers of the subject in both the elementary and high school grades, and also to take care of the emergency defense needs:

A. *Beginning with the 1942 Spring Semester*

In order to meet the situation as effectively as possible during the immediate emergency, a suggestive curriculum of mathematics courses is offered as a minimum training for those who wish to teach the subject in the elementary grades (1 to 6 inclusive), and a curriculum of courses for those who are to teach in the secondary division (grades 7 to 12 inclusive). To meet the present emergency, these suggestive curricula are intended to apply particularly to our Sophomore and Junior students who have not already completed, or who will not have completed, two full years of college mathematics by the close of the 1941 summer session.

1. *Curriculum for Elementary Teachers*

Math 116. *Fundamentals of Arithmetic.*—Three hours.

Math 114. *Solid Geometry* (*Review of Plane Geometry*).—Three hours.

Math 111. *College Algebra* (*Especial attention to fundamentals.*)—Three hours.

Math 112. *Plane Trigonometry* (*Especially numerical trig.*).—Three hours.

Math 370. *Materials and Methods of Arithmetic.*—Three hours.

2. *Curriculum for High School Teachers*

Math 111. *College Algebra.*—Three hours.

Math 112. *Plane Trigonometry.*—Three hours.

Math 114. *Solid Geometry.*—Three hours.

Math 211. *Plane Analytic Geometry.*—Three hours.

Math 212. *Solid Analytics and Introduction to Calculus.*—Three hours.

Math 401. *Materials and Methods of Secondary Mathematics.*—Three hours.

Curriculum 1 may be taken up by even those Sophomore and Junior students who have no credits in college mathematics, provided their plans and present schedules make it convenient and possible to do so, doubling on the subject of mathematics during our spring and summer semesters, if necessary.

Curriculum 2 may be taken by any Sophomore or Junior who has completed as much as one year of college mathematics, or who has credits of two units of high school algebra and one unit of plane geometry.

B. *Beginning with the 1942 Fall Semester*

Beginning with the 1942 fall semester, it is recommended that one full year of mathematics be required for a bachelor's degree from a Texas teachers college; the work to be offered under the following plan:

For Group One

The work for this division to be suited to the needs of students with at least three units of high school mathematics, including two years of algebra and one of plane geometry. The materials for this Freshman will necessitate no material variation from that usually given in the college Freshman year—three semester-hours of college algebra and three semester-hours of plane trigonometry and introduction to coordinates and graphs.

For Group Two

The work for this division to be suited to the needs of students with less than three units of high school mathematics, and to include thorough study, drill, and applications of the fundamental facts, principles and processes of arithmetic, algebra, plane and solid geometry, and numerical trigonometry.

"To date, March 31, 1944.—The above suggestive solution for teacher shortage for mathematics, proposed more than three years ago, has met with limited approval only. The two types of mathematical training for elementary and secondary teachers of mathematics are now being offered in the East Texas State Teachers College; however, mathematics remains entirely on the elective basis for any one of our four degrees,—the B.S., B.A., M.S., M.A. In my judgment, no program of training can be made effective unless and until work is definitely prescribed."

Tune: *The Wearing of the Green*

1. Oh *X* how I do loathe you
You slippery old unknown,
I'm sure you hold the record
For the sorrows you have sown.
I seek you in the darkness
That forms my mental night,
And when I think I have you
You vanish from my sight.

2. If this is this and that is that
Pray what is so and so?
The teacher asks me every day
I'm sure I never know.
She thinks I'm very stupid
But I am not to blame.
It's you, you skulking creature, who
Won't answer to your name.

3. Most valiantly I hunt you out
In every likely lair,
But when I catch a glimpse of you
You wink and disappear.
With wings upon your shoulders
And heels awinged too,
Alas, can I, poor mortal,
Expect to cope with you?

4. Your heart is full of mischief
Brewing troubles by the peck,
If ever I do corner you
Be sure I'll wring your neck.
But *X* I want to warn you
You really must beware,
Or you'll never get to Heaven
For there are no secrets there.

Pitfalls in the Trigonometric Solution of an Oblique Triangle

By Irwin M. Rothman

Manhattan High School of Aviation Trades, New York City

Introduction

Textbooks in plane trigonometry generally treat the solution of oblique triangles by the Law of Sines, Law of Cosines, etc. However, if one uses some of these laws without a complete understanding of their limitations, incorrect results are often obtained. The only case which most textbooks treat adequately in this respect is the solution of a triangle in which two sides and an angle opposite one of them are given, usually referred to as the "Ambiguous Case." However, in other cases, such as the one in which two sides and the included angle are given, or the one in which the three sides are given, care must be taken if incorrect solutions are to be avoided.

Purpose of Article

The purpose of the present article is to show some of the "pitfalls" involved in the use of various laws, and how they can be avoided by following certain procedures. Without an adequate knowledge of these procedures, incorrect solutions may be obtained, even though the correct laws may be used and no errors of computation are made.

Two Sides and Included Angle Are Given

Let us consider the solution of a triangle when two sides and the included angle are given (side, angle, side). This case has many applications in physics (the parallelogram of forces and other vector problems) and in air navigation (the wind-triangle of velocities). Below is a problem involving the parallelogram of forces, and a "solution" as given by a student who employs the Law of Cosines followed by the Law of Sines.

Example: Two forces, 11.00 lb. and 21.00 lb. respectively, act at an angle of 130°. Find the magnitude and direction of the resultant.

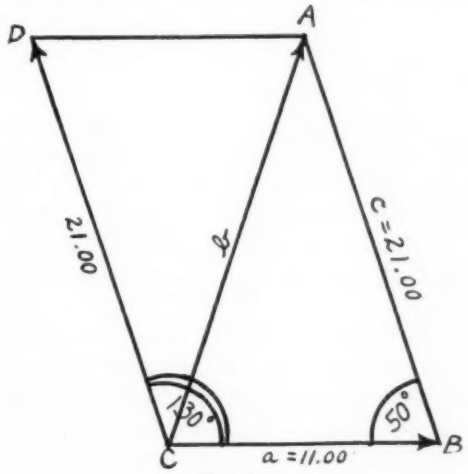

Fig. 1

Given: Two forces, $CB = 11.00$ lb. and $CD = 21.00$ lb., acting at an angle of 130° ($\measuredangle BCD = 130°$).
Find: Magnitude and direction of the resultant CA.
Solution:

$$\measuredangle B = 180° - 130° = 50°.$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 121 + 441 - 462\ (.6428)$$

$$= 562 - 296.97$$

$$= 265.03$$

$$\therefore\ b = \underline{\underline{16.28}}$$

$$\frac{\sin ACB}{c} = \frac{\sin B}{b}$$

$$\sin ACB = \frac{c \sin B}{b}$$

$$= \frac{21 \sin 50°}{16.28} = \frac{21(.7660)}{16.28}$$

$$= \frac{16.0860}{16.28}$$

$$= .9881$$

$$\therefore\ \measuredangle ACB = \underline{\underline{81°10'}}$$

∴ The magnitude of the resultant is 16.28 lb., and the direction is 81° 10′ from the 11.00 lb. force.

The above solution seems correct, at first glance. However, if we attempt to find $\sphericalangle CAB$ by the Law of Sines, we obtain:

$$\frac{\sin CAB}{a}=\frac{\sin B}{b}$$

$$\sin CAB=\frac{a\sin B}{b}$$

$$=\frac{11\sin 50^\circ}{16.28}=\frac{11(.7660)}{16.28}$$

$$=\frac{8.4260}{16.28}$$

$$=.5176$$

$$\therefore\quad \sphericalangle CAB=31^\circ 10'$$

We find that the sum of the three angles is 31° 10′+50°+81° 10′, or 162° 20′, and not 180°. Therefore, there must be some error in the work.

The fallacy here is similar to several well-known fallacies involving the Law of Sines.

In finding $\sphericalangle ACB$, from $\sin ACB=.9881$, the result 81° 10′ was obtained. However, the result could also have been the supplement of 81° 10′, or 98° 50′, because $\sin(180-x)=\sin x$. In fact, the correct answer for $\sphericalangle ACB$ is 98° 50′.

In the above example, a *scale* drawing, using protractor and ruler (or coordinate paper) would show that $\sphericalangle ACB$ is an obtuse angle, and not the acute angle shown in the diagram. However, if the result were close to 90°, such as 89° or 91°, a scale drawing would probably be insufficient.

Procedures to Follow when "Side Angle, Side" Are Given

In order to avoid the "pitfall" described above, any one of the following procedures could be followed:

1. After finding the third side by the Law of Cosines, use the Law of Sines to find the *smaller* of the two unknown angles, i.e., find $\sphericalangle CAB$ first. This angle can not be an obtuse angle, and the "pitfall" will be avoided. Angle ACB could then be obtained by subtracting the sum of the other two angles of triangle ABC from 180°.

2. In examples of this type, draw the diagram so that the *larger* of the two given forces is in a horizontal position, and find the *direction* or *angle* which this larger force makes with the resultant, or $\sphericalangle ACD$. This angle is an acute angle (See Figure 2).

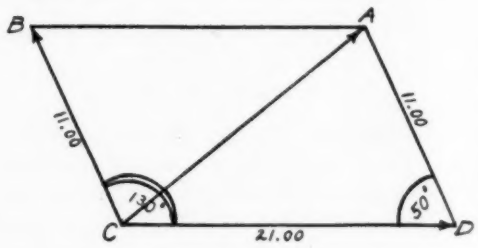

Fig. 2

3. Draw a *scale drawing* of the parallelogram of forces, or the triangle involved. As mentioned above, this would not be of much assistance when the angle to be found is close to 90°, e.g., 89° or 91°.

4. Of course, the "pitfall" could be avoided by using the alternative method, i.e., first using the Law of Tangents in order to find the two unknown angles of the triangle, and *then* using the Law of Sines to get the magnitude of the resultant. However, this method may be more cumbersome than the Law of Cosines method, especially when the given forces have only one or two significant figures.

5. Another method for avoiding the "pitfall" would be to find side b by the Law of Cosines, as done in the solution shown, and then find $\sphericalangle ACB$ by using the formula

$$\cos\sphericalangle ACB=\frac{a^2+b^2-c^2}{2ab}.$$

Procedures to Follow when "Side, Side, Side" Are Given

In solving a triangle when the three sides are given (side, side, side), the same difficulties mentioned previously in this article may arise. Unless proper precautions are taken, the results obtained may be incorrect, even though no errors of computation are made.

The procedures to follow in order to avoid making serious errors in finding the three angles of a triangle are as follows:

Example 1. If $a=50$, $b=60$, $c=80$, find the three angles.

In this example, if the Law of Cosines is to be used, find the *largest* angle first, angle C. If $\cos C$ is *negative* (it is, in this case), angle C is an obtuse angle. Angles A and B may then be obtained by the Law of Sines, without any difficulty, because neither A nor B can be obtuse. Alternatively, angle A may be found by the Law of Sines, and angle B obtained by subtracting $(A+C)$ from 180°.

Example 2. If $a=31.65$, $b=42.81$, $c=54.78$, find the three angles.

Three methods are usually employed in solving this example. These methods involve formulas for the sine, cosine, or tangent of the half-angles of a triangle.

1. If the sine of the half-angle is used, find the *largest* angle first, angle C. This is the only angle that may be obtuse. The formula to be used is

$$\sin\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}.$$

After finding $\sphericalangle C$, the Law of Sines may be used to obtain $\sphericalangle A$ and $\sphericalangle B$, or else $\sphericalangle A$ could be found by the Law of Sines, and $\sphericalangle B$ obtained by subtracting $(A+C)$ from 180°.

2. If the cosine of the half-angle is used, find the *largest* angle first, angle C. The formula to be used is

$$\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}.$$

The same procedure is followed as in 1., above.

3. If the tangent of the half-angle is used, it is best to find $\sphericalangle C$, $\sphericalangle A$, and $\sphericalangle B$ without using the Law of Sines. The formulas to be employed are:

$$\tan\frac{A}{2}=\frac{r}{s-a},\qquad \tan\frac{B}{2}=\frac{r}{s-b},$$

and

$$\tan\frac{C}{2}=\frac{r}{s-c},\quad \text{where}$$

$$r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Reviewing the procedure for finding the three angles when the three sides are given, the best method is to find all three angles by the tangent of the half-angle formulas, because no difficulties in the Law of Sines can arise.

If any other method is used, the *largest* angle (opposite the largest side) should be found first, and then the Law of Sines can be applied without any difficulty.

Conclusion

Thus we see that whenever, in the course of a problem, the Law of Sines is used to determine an angle of a triangle (as in the formula

$$\sin C=\frac{c\sin B}{b},$$

the solution may be an example of the "Ambiguous Case," *no matter what the original given data may have been.* This is true because in the above formula two sides, c and b, and an angle opposite one of them, angle B, are used; the fact that the original problem had other data given, as two sides and the included angle, or three sides, does not alter the situation in any way. Consequently, whenever the sine of an angle is found, the angle may be an acute angle or its supplementary obtuse angle. Proper procedures have been described in this article, so that the correct angle will be obtained in each case.

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◆ THE ART OF TEACHING ◆

Functioning Mathematics

By SAMUEL I. ALTWERGER
Bronx Vocational High School, New York, N. Y.

HIGH SCHOOL students and graduates in increasing numbers read technical manuals, pamphlets, periodicals, and handbooks. Much of this material contains a substantial amount of mathematics and references to mathematics. Yet the mathematical education, even a four years education, in the secondary school has been and is insufficient to meet their needs. The war has brought this sharply to our attention.

A high school graduate has a right to expect that his training will cast some light, if not insight, on his technical reading. Take a young man looking through a radio manual or text. It is replete with graphs and formulas. He has studied both, yet frequently he is in a strange world. Too often he takes a refresher course and he gets again the same material that he has found of little avail. There is something the technical world does with graphs, formulas, ratios, equations, and the like that he was not taught to do or understand.

Is it not our obligation, as mathematics teachers, to prepare our students to understand the mathematical aspects of such reading? It is not sufficient to say that we teach pure mathematics. What makes our mathematics pure? Is it (what we choose to teach) its aloofness from economic, social, or technical value? And how shall we justify a great many topics in the present course of study that have little mathematical value in the sense that they hardly ever recur in or are requisite for more advanced mathematical studies? It is time we set up a test for the selection or retention of material. Such a test might include for example such questions as the following:

1. Does this material contribute to the understanding of significant mathematics just ahead? 2. Does it contribute to the understanding of significant social, economic, scientific, and technical methods and concepts involving mathematics and mathematical thinking?

Let us not merely speak of the power of mathematics. Let the power of mathematics speak for itself through numerous and diverse applications. In such an atmosphere the student will feel that he is working and thinking in a real world. Too often we are content to assure him of the place of mathematics in our society, leaving it to him to discover how it functions. As a result a large portion of our population except for the war, would have been convinced of the futility of a mathematical education. This is almost paradoxical, at a time when mathematics actually functions to a greater degree than ever before. In any study and analysis of current technical literature, intended for high school graduates, it will be evident that there is a wealth of material, providing that we do not make the approach with preconceived notions of what should be there or seek merely applications for that which is being taught. Herein lies the opportunity to reflect in the classroom some of the culture of the day.

This article will suggest certain activities in connection with graphs, to take but one example, which because of their wide use and because of their intrinsic mathematical value ought to occupy a place in the course of study. Most of them are not being taught. Instead, in the upper grades we fool around with intersecting circles and lines and roots of quadratics,

and the like, which cast little light on anything but themselves.

Let us begin with linear graphs. Actual practice indicates the need of broadening the notion of variation to include range and the sense of variation. Thus, in Figs. 1 and 2, while x varies from a to b, y varies from c to d in the same sense. In Fig. 3 the variation is in the opposite sense. In the first case the range of a to b equals that of c to d, which is not so in the next two cases. If x varies at a constant rate in the interval, y will vary at the same, lesser, and greater rates repectively in the three

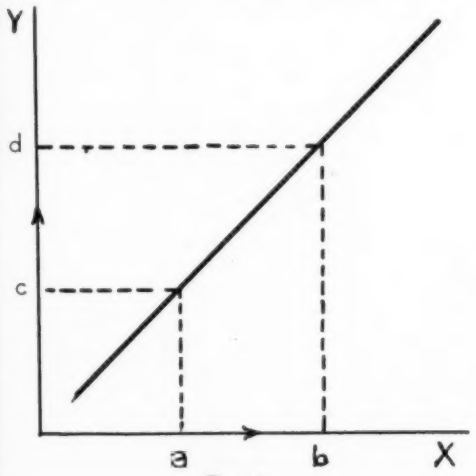

FIG. 1

cases. If x varies harmonically y will do likewise although the rate and range may be different. In any case the rate of change will depend on the ratio of cd to ab, which of course is the slope.

Through abundant illustrations drawn from diverse fields, these notions, which play an important part in interpreting and understanding relationships, can be imparted. It is advisable that such applications precede to some extent any attempt at abstract discussion or generalization of the kind given here. The beginning could be made in such concrete terms as time, altitude, expansion, temperature, solubility, pressure, and the like. This manner of interpreting a graph may easily be extended to non-linear curves, experimental or theoretical. Such analysis need not and should not be restricted only to those graphs actually constructed by the students. Studies of this kind, for example,

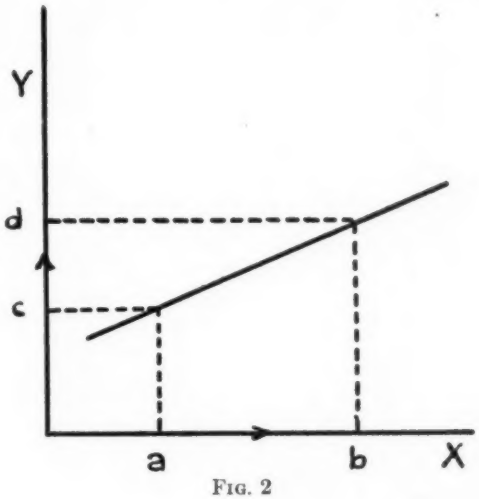

FIG. 2

are basic in radio to help explain different actions of radio tubes as illustrated in Figs. 4 and 5. In the former the grid potential varies from a to b in an harmonic manner as indicated by the curve directly

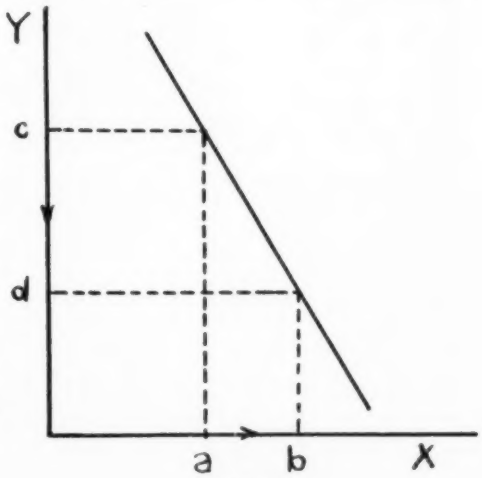

FIG. 3

beneath it. The plate current will vary in a corresponding manner over the range c to d. The center point M of ab will correspond to the point M', which is considerably below the center because of the curvature of the graph. Therefore, the variation from M to a corresponds to the variation from

M' to c. If $M'c$ is made quite small, by moving a to the left, the variation within $M'c$ practically disappears. This is essentially the requirement of a detector tube. The negative variation is virtually eliminated.

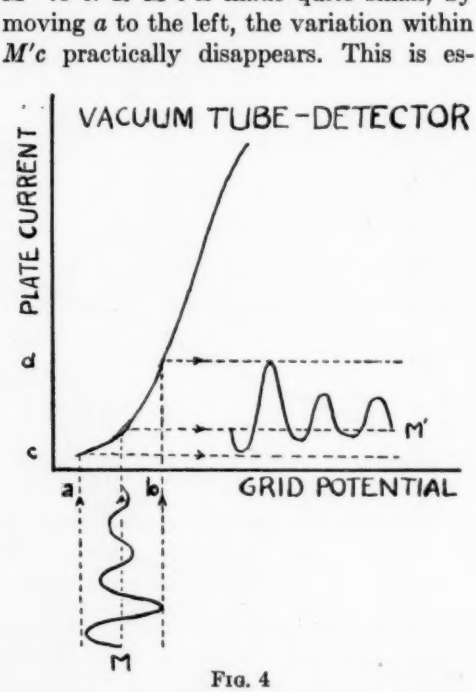

FIG. 4

In Fig. 5 there is no elimination of one end of a variation. The need is to amplify the whole variation, that is, to increase the resulting range. Therefore a is moved to the right, by using a higher grid potential, and the complete harmonic variation is amplified. Thus, while the tube is the same in Figs. 4 and 5, yielding the same "characteristic" curve, the action of the tube depends on the range of the grid potential that is used.

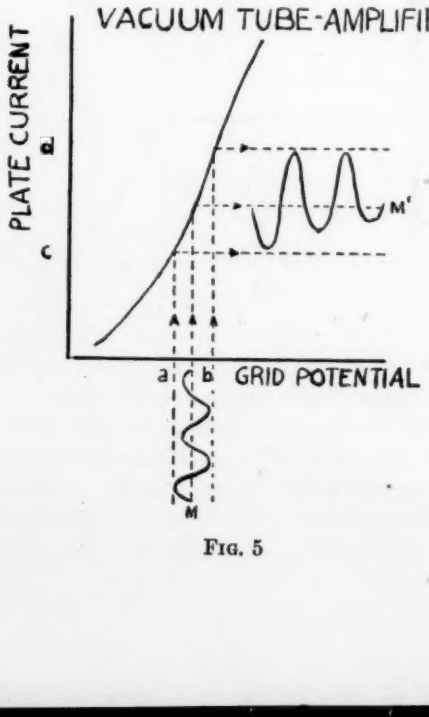

FIG. 5

Fig. 6 illustrates the graphical derivation of an empirical formula. The experimental values are plotted as shown. These seem to cluster about a straight line. This line can be drawn in at sight and a fair equation obtained from a knowledge of slope and intercept. The sum of the distances of the points below the line ought to be equal to the distances of the points above the lines. This can be done at sight fairly well by matching a point below with one above. The values 0.25 and 1.29 are obtained directly from the graph. (The average error may be obtained by comparing the calculated values of T from this formula with the experimental values.) A little practice will lead to good results. It is suggested that some simple experiments be performed and the equations obtained in this manner. In addition, data for such work is obtainable from many sources. The effectiveness of the formula as a means of prediction can then be demonstrated. Questions of accuracy,

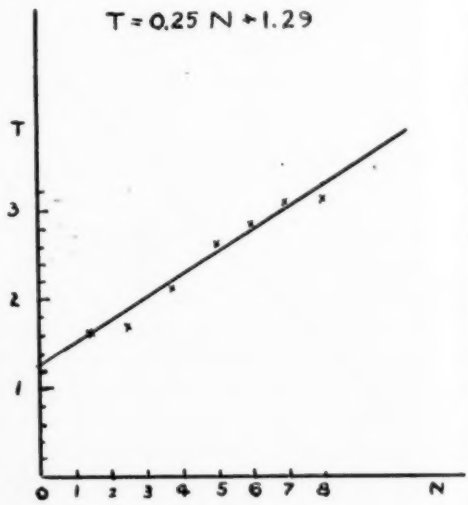

FIG. 6

range, and variation will of necessity enter into these discussions. It is likely that this work can help gain an insight into the experimental method. (Hooke's Law, circumference of a circle, Centigrade and Fahrenheit temperatures, Ohm's Law, and electrical resistance of a wire are some possible experiments.)

Figures 7 and 8 suggest other applications of the linear graph—families or pencils of straight lines, each one of which is obtained by assigning some value to one of

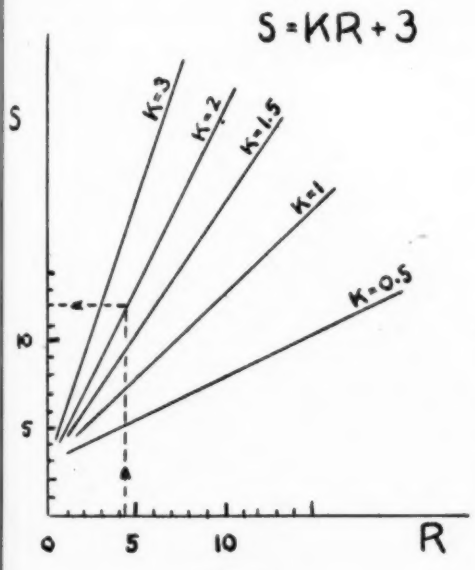

FIG. 7

the variables, as with K and T below, and graphing the resulting linear equation. The process is then repeated by assigning new values to the first variable. This type is coming into widespread use for the purpose of obtaining quickly and efficiently sets of values that may be related in this manner. For example, when $R=4.5$ and $K=2$ the value of $S=12$ is found by following the dotted line. Questions concerning variation are easily studied by means of the graph. It can be seen at a glance that any change in R will cause increasingly larger changes in S as K increases. This, and related facts can be referred to the graph for specific values or ranges. The alternative to this graph is either a long and cumbersome table or frequent computations. Such graphs are being published for ready reference. They are easy of construction requir-

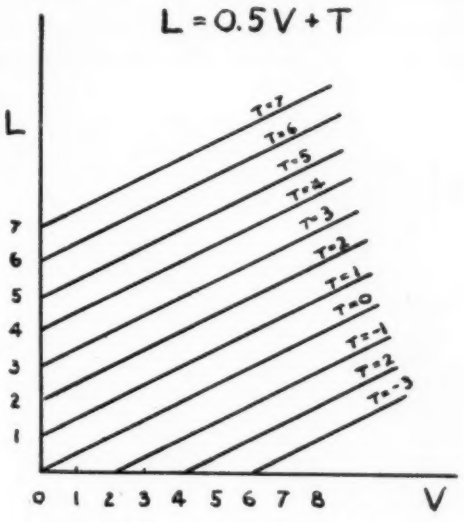

FIG. 8

ing again only a knowledge of slope and intercept. It should be noted that it is immaterial from the viewpoint of the graph which variable receives the preassigned values. A particular illustration, however, may dictate a desirable choice. A simple and effective illustration of Fig. 7 is the rate-time-distance graph printed in the Civil Aeronautics Bulletin on Navigation. The same publication contains an illustration of Fig. 8 in finding the true air speed.

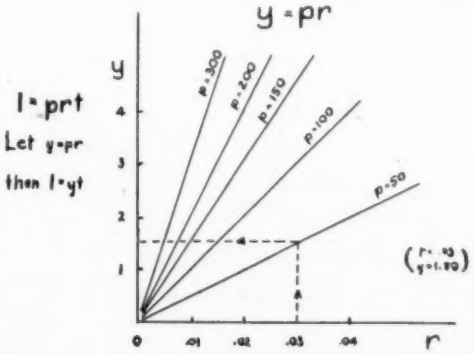

FIG. 9

These two illustrations offer convincing evidence of the value of this graphic method.

Figures 9, 10, and 11 show in some detail the construction of another device which is extensively used to obtain values in four-variable relationships. The simple case for the interest formula is illustrated. By the introduction of a parameter (y) the

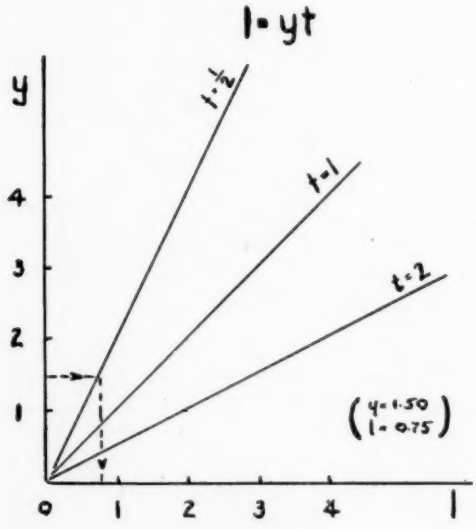

FIG. 10

original formula is broken down to two three-variable relationships each of which is graphed in the manner of Fig. 7. The parameter may then be eliminated by

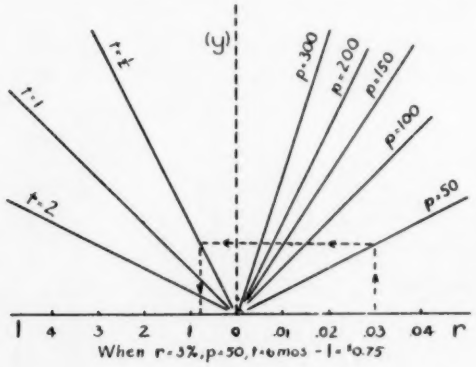

FIG. 11

joining the two graphs as in Fig. 11. It is clear from the latter that the value of y, and therefore the y axis, is no longer necessary. Fig. 12 shows a way of joining the two graphs in a space-conserving manner, which is the way usually employed. To obtain this, Fig. 10 is turned over and placed so that it forms a rectangle with Fig. 9. For specific values of r, p, and t the interest I is easily obtained by following the arrows as indicated in each drawing. The final set of arrows in the last drawing being merely a composite of the first two. It should be obvious too that the unknown may be any one of the variables. Science and industry are printing many of this form for general use. All that is really needed is specific instruction in reading, consisting merely of directions of motion. The significant feature is that this

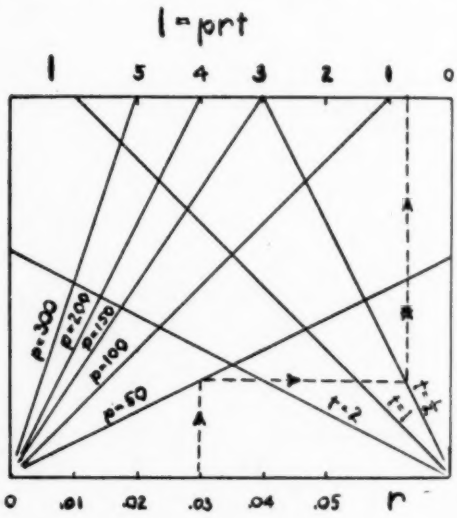

FIG. 12

type of graph can be used for any four-variable function. Each of the families takes care of one variable and the two axes take care of two more. This device can be used for such formulas as the volume of a rectangular solid, Boyle's Law, total force on a surface submerged in a liquid and so on. By permitting one or both of the sets to be curved lines, as in the graphs immediately following, the scope of this method may be extended indefinitely.

Figures 13, 14, and 15 present families of quadratics. These types are used to obtain specific values in three-variable func-

tions as well as to study the related effects of the variables. They are in common use in many fields and numerous applications are available. They play for example a significant role as characteristic curves in radio. The recent Army and Navy technical manuals employ a number of these also.

Graphic study and analysis need not be restricted only to those cases actually graphed by the students. Opportunities

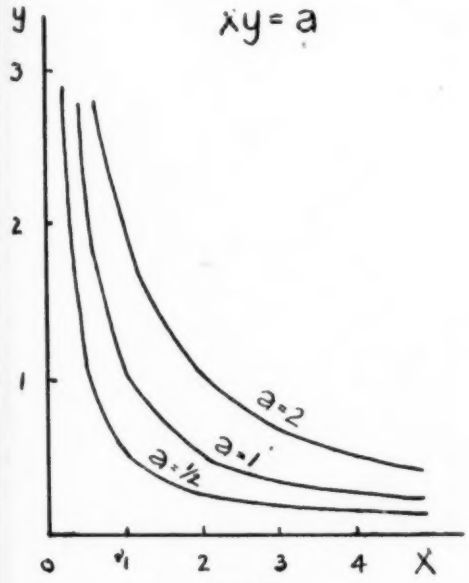

FIG. 13

for reading and study must be provided. This will necessitate mimeographed graphs containing some background information and appropriate questions. This aspect of the work may be extended to more advanced cases than those mentioned here. A study of a graph of average plate characteristics for radio tubes, for example, would give as much information, and with greater clarity, than the reading of a full chapter on the subject. To take another example, a similar exercise would give a clear and quick grasp of the effects of flaps on the lift coefficient for airplanes. Many other instances are obtainable within the realm of the student's interests.

Wherever applications are used some discussion of the meaning or significance of the setting is of great value. This will

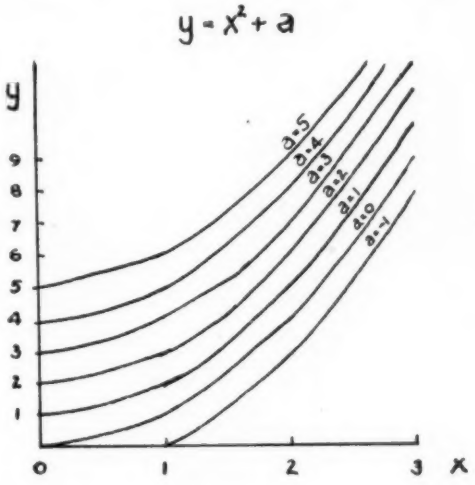

FIG. 14

lead to an appreciation of the role of mathematics in various fields. If the students, or even if only one student, can say something about the application a certain

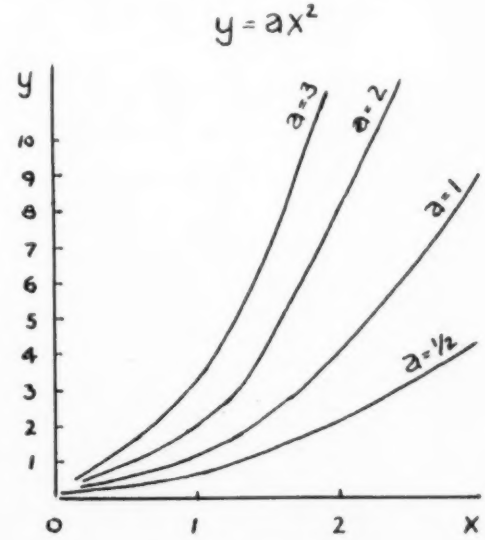

FIG. 15

degree of reality is added to the work. The psychological and pedagogical value of such "digressions" cannot be overestimated. Too often we are loathe to digress

missing valuable opportunities to demonstrate the power and utility of the subject. In fact, even a casual verbal reference is better than none at all. The teacher need not shun this because of lack of familiarity with the application as no thorough acquaintanceship is really necessary. In a sense, we need only cultivate the technique of the librarian; know sources of reference and have some familiarity with the terminology.

Figures 16, 17, and 18 introduce the alignment charts. These are coming into fairly extensive use as convenient means of obtaining related values from intricate formulas. To obtain results one need only place a ruler across the scales, as indicated by the dotted lines. In Fig. 16 the dotted line completes a trapezoid in which a portion of the y-axis becomes a median, giving rise to the formula indicated. Thus, this chart can be used to obtain averages quickly. By the use of half, double, triple, square root, cube root, or logarithmic scales the same graph can be used to serve a host of other formulas. The formula for Fig. 17 is evident by studying the similar triangles formed by the dotted lines. Specific values or an unknown value in formulas of this type are quickly obtained by placing a

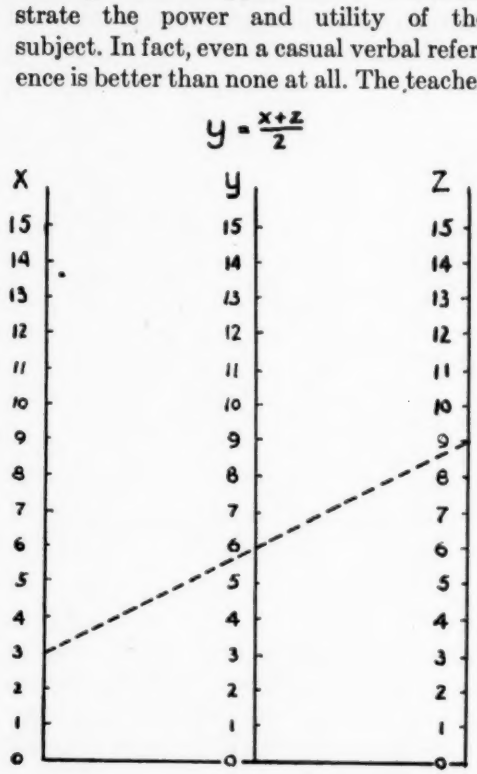

Fig. 16

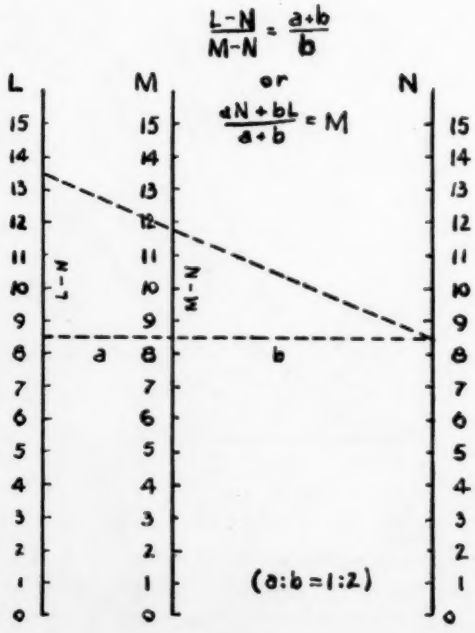

Fig. 17

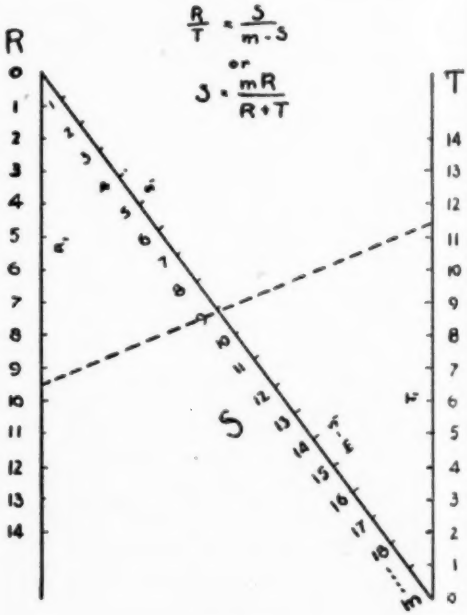

Fig. 18

straight edge across the chart. Again, by varying the nature of the scales, a great many variations are possible in the formula. Fig. 18 is also explained by means of similar triangles.

Using the equally spaced axes of Fig. 16 and applying a logarithmic scale to each, by the method mentioned below, we get the illustration of Fig. 23. Substituting $\log q$ for x, $\log r$ for z and $\log p$ for y in the original formula of Fig. 16, we get $\log p = \frac{1}{2}(\log q + \log r)$. From this we get that $2 \log p = \log q + \log r$. This in turn leads to the formula indicated on the graph of Fig. 23. Now, if it is desired to find the value of p when $q=2$ and $r=8$, we need only place a straight edge across these values as indicated by the dotted line and read the value

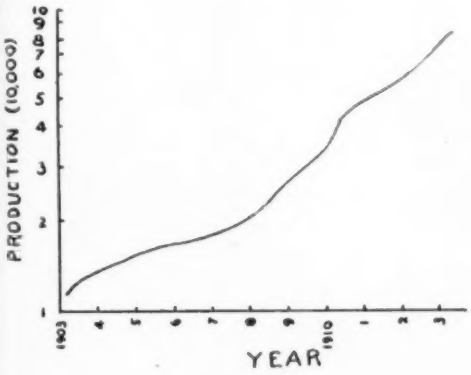

Fig. 19

of p as 4. Similarly, given any two values for any two of the variables the third may be found. Additional subdivisions on the scales would of course be desirable for accurate results.

Fig. 19 illustrates the use of a semi-logarithmic graph. One axis has an arithmetic scale while the other has a logarithmic scale. The latter may be obtained from a slide rule and is easily expanded or contracted as shown in Fig. 21 (by applying a geometry theorem). The logarithmic scale reflects the rate of change of the variable rather than its arithmetic size. For this reason, this graph is extensively used for studies in growth, development, or analyses of trends. Illustrative material may be found in business and industrial publications. Physical science publications may

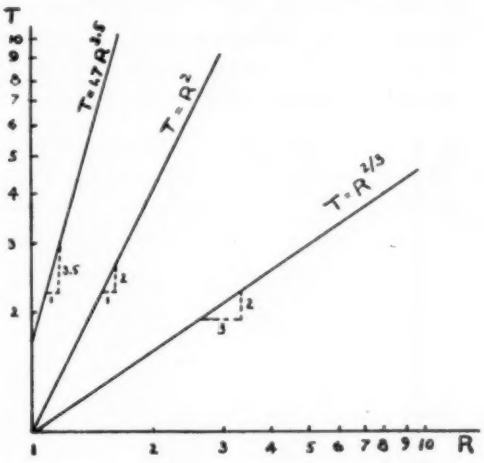

Fig. 20

also be used as a source of material especially where studies in growth or deterioration are made. This method is also applied to experimental data to detect exponential relationships. For, if x is a function of the logarithm of y, it follows

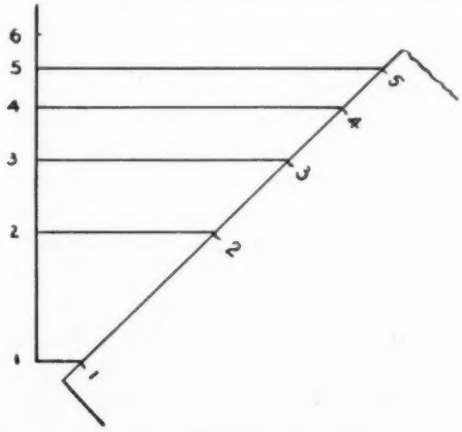

Fig. 21

that y is an exponential function of x. In fact, if the data is linear, or nearly so, the equation is quickly obtainable by the method outlined in connection with Fig. 6.

Fig. 20 shows a logarithmic graph, both scales being logarithmic. The illustrations show that quite advanced curves may be

made linear. This is because the power formula $y = kx^n$, when taken logarithmically becomes $\log y = \log k + n \log x$ so that n is the slope and $\log k$ the intercept of a straight line. The convenience is obvious. Where k and n are empirical values this method is almost mandatory. Further, by

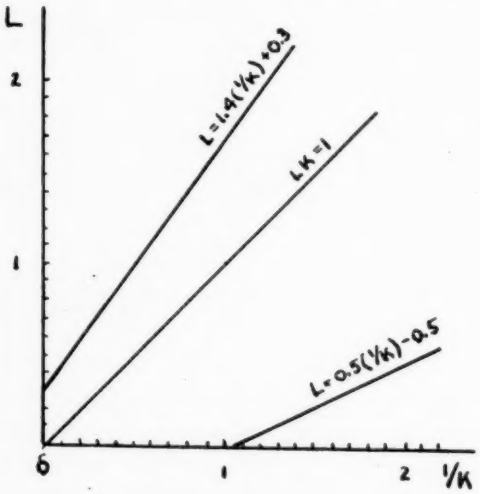

FIG. 22

applying the method of Fig. 6, such graphs are used to predict and obtain formulas from experimental values. Illustrative material for this type is also found in the type of sources already mentioned. Once the scales are put down the actual graphing is no different than the usual cases. Reference to logarithm tables is unnecessary.

Finally, Fig. 22 is suggestive of another host of applications where one or both of the axes refer to a function of the variable, in this case $1/k$. This practice offers many advantages for problem analysis and other studies. Frequently, one or both of the axes, as is often done in the service manuals mentioned, is taken as a function of more than one variable.

It is probable that the material above may appear too difficult. For those unfamiliar with these graphs some reflection and perhaps reading is necessary. Only a brief description is possible in this space. It is likely that the tremendous impact of reality through truly functioning mathematics, of glimpses of mathematical methods of study, analysis, and formulation, of contact with some aspects of the scientific method, and of glimpses of business, science, and industry at work with mathematics will arouse a keen interest and help to offset some of the difficulties. Of course, much of this, being experimental, will take some time to develop and organize for most effective presentation.

It is obvious too that similar developments ought to be made in connection with many other topics—notably the equation, the formula, trigonometry, and

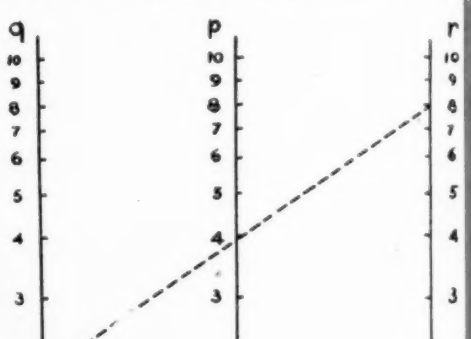

FIG. 23

many aspects of geometry. It will be a simple matter then to move out the obsolete, much of which is a hand-me-down of someone's mathematical playfulness or, at best, a response to a need no longer existent. This type of work is mathematics and there is plenty of it to take the place of the desirable deletions.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
Midwood High School, Brooklyn 10, New York

The American Mathematical Monthly

June–July, 1944, Vol. 51, No. 6.

1. Wilder, R. L., "The Nature of Mathematical Proof," pp. 309–323.
2. Eves, Howard, "Concerning Some Perspective Triangles," pp. 324–331.
3. Mordell, L. J., "Rational Points on Cubic Curves and Surfaces," pp. 332–339.
4. Brenner, Joel, "Determination of Latitude in an Emergency," pp. 343–344.
5. Eves, Howard, "A Note on Envelopes," p. 344.
6. MacDuffee, C. C., "On the Concept of Divisor," pp. 345–346.
7. Craig, H. V., "A Vectorial Development of Two Differentiation Formulas," p. 347.
8. War Information: The Teacher Problem in Secondary Mathematics; Eastern Conference on Navy V-12 Programs; Some Data on Mathematics in Colleges; Mathematical Publication During Wartime; Notes on Post-War Education; pp. 359–365.

National Mathematics Magazine

May 1944, Vol. 18, No. 8.

1. Kasner, Edward, and Kalish, Aida, "The Geometry of the Circular Horn Triangle," pp. 299–304.
2. Sleight, E. R., "Mathematics in Scotland before the Eighteenth Century," pp. 305–314.
3. Bennett, Albert A., Trends in the Teaching of Secondary School Mathematics," pp. 315–322.
4. Boyer, Carl B., "Zero: The Symbol, The Concept, The Number," pp. 323–330.

School Science and Mathematics

June, 1944, Vol. 44, No. 6.

1. Lippitt, Vernon G., "The Teaching of Mathematics as Viewed by an Engineer," pp. 505–510.
2. "Science and Mathematics in Educational Programs for Returning Service Men and Women," pp. 517–520.
3. Terracini, Alejandro, "Words and Things; Prolate and Curtate Cycloids," pp. 526–529.
4. Moody, F. W., "Approximations in Physics Teaching," pp. 536–538.
5. Jerbert, A. R., "Mathematics—A Cultural Subject," pp. 541–544.
6. Porges, Arthur, "Again That Quadratic Equation," pp. 565–568.
7. Nyberg, Joseph, "Notes from a Mathematics Classroom" (continued), pp. 569–572.
8. Larsen, Harold D., "A Note on Long Division," p. 578.

Scripta Mathematica

September, 1943, Vol. 9, No. 3.

1. Shaw, James Byrnie, "Ocult Symmetry," pp. 129–138.
2. Young, Anne, "Root-Five" (a poem), p. 138.
3. Karpinski, Louis C., "The Progress of the Copernican Theory," pp. 139–154.
4. Whitlock, W. P., Jr., "Rational Right Triangles with Equal Areas," pp. 155–161.
5. Fraenkel, A. A., "Problems and Methods in Modern Mathematics," pp. 162–168.
6. Richards, John F. C., "A New Manuscript of a Rithmomachia," pp. 169–183.
7. Curiosa, pp. 189–190.
8. Resnikoff, Louis A., "Jewish Calendar Calculations," pp. 191–195.
9. Worrell, W. H., "An Interesting Collection," pp. 195–196.
10. Plotnick, S. M., "The Sum of n Terms of the Fibonacci Series," p. 197.

Miscellaneous

1. Armstrong, H. C., "Place of Vocabulary in Mathematics," *California Journal of Elementary Education*, 12: 200–206, May, 1944.
2. Ayer, F. E., "Analytic Geometry," *Journal of Engineering Education*, 34: 525, April, 1944.
3. Barclay, E., "Arithmetic Can Be Fun," *Texas Outlook*, 28: 23–24, April, 1944.
4. Baxter, B., "Arithmetic Program in the Elementary School," *California Journal of Elementary Education*, 12: 233–239, May, 1944.
5. Brueckner, L. J., "Social Arithmetic," *Instructor*, 53: 17–18+, September, 1944.
6. Freeman, F. N., "Teaching Mathematics for the Million," *California Journal of Secondary Education*, 19: 246–254, May, 1944.
7. Grossnickle, F. E., "Two Kinds of Problems," *Instructor*, 53: 22, June, 1944.
8. Miller, G. A., "Mathematics in a Nutshell; Objection to Brief Text-books," *Science*, 100: 31, July 14, 1944.
9. Snyder, M. E., "When Shall We Begin Number Work?" *Grade Teacher*, 62: 30+, September, 1944.
10. Zant, J. H., "Role of Mathematics in Postwar Education," *Phi Delta Kappan*, 26: 62–64, January, 1944.

NEWS NOTES

The American Council on Education, acting in conjunction with the Canada and Newfoundland Education Association, the Canadian Teachers' Federation, and the National Conference of Canadian Universities, announces the establishment of a joint *Canada-United States Committee on Education.* The Committee seeks to provide opportunity for consultation among educational leaders and associations of the two countries and to aid in developing educational programs for strengthening the respect and understanding which citizens of each country now have for the other. The Committee is nongovernmental in character; it is a pioneering agency for cooperation among educational groups in two friendly countries which are alike concerned with adjustments in education needed to meet the urgent international problems of the postwar period.

Members of the Committee from the United States are: J. W. Brouillette, Director of General Extension, Louisiana State University; J. B. Edmonson, Dean, School of Education, University of Michigan; Arthur A. Hauck, President, University of Maine; Ernest Horn, Professor of Education, University of Iowa; Erling M. Hunt, Professor of History, Teachers College, Columbia University; Howard E. Wilson, Professor of Education, Harvard University; Carl Wittke, Dean of Liberal Arts, Oberlin College; and, *ex officio*, Herman B. Wells, President, University of Indiana, and George F. Zook, President, American Council on Education. From Canada the Committee members are: Victor Doré, Superintendent of Education, Province of Quebec; C. C. Goldring, Superintendent of Schools, Toronto; M. E. LaZerte, Dean, Faculty of Education, University of Alberta; Edouard Montpetit, Dean, School of Social Sciences, University of Montreal; Fletcher Peacock, Director of Education, Province of New Brunswick; Charles E. Phillips, Professor, History of Education, Ontario College of Education; Reginald G. Trotter, Professor of History, Queen's University; and, *ex officio*, V. K. Greer, President of the Canada and Newfoundland Education Association; James S. Thomson, President of the National Conference of Canadian Universities; and E. Floyd Willoughby, President of the Canadian Teachers' Federation. An Executive Group for the Committee is made up of its co-Chairmen, Dean Edmonson and Dr. Peacock, and its co-Secretaries, Dr. Phillips and Dr. Wilson.

The first meeting of the Committee was held in Niagara Falls, Ontario, on September 18–20, 1944. Among the guests at the meetings were James T. Shotwell and Malcolm W. Davis of the Carnegie Endowment for International Peace. During its sessions the Committee tendered Dr. Shotwell a dinner in tribute to his years of service in developing the scholarly foundations for a program of intercultural relations between the United States and Canada. Mr. Kenneth Lindsay, member of the British Parliament and formerly Under-Secretary in the British Board of Education, was also a guest of the Committee. He reported to it on recent educational developments in Great Britain, and on activities among the United Nations for educational advance during the postwar period.

At the Niagara Falls meeting a statement was adopted (shortly to be issued in pamphlet form by the Committee) which reviews the similarities, the differences, and the mutual interests of the two countries, and suggests the function of education in perpetuating the goodwill which now exists. The following paragraphs from that statement indicate the point of view and general policy of the joint Committee.

"Though present happy relations between Canada and the United States are firmly established, it would be unwise in the extreme to take their continuance for granted and to neglect the good offices of friendship. Cooperation can never be regarded as a *fait accompli;* it can endure only as an active, on-going process. . . . The history of developing friendship between our two countries gives us good reasons for congratulation and pride but shows no less clearly the need for keeping the basis of that friendship strong. During more than a century of peace between Canada and the United States there have been few decades without occasion for disagreement; there will be such occasions again. Only a living and flourishing friendship secured by deep-spreading roots of respect and understanding can prevent occasions of difference from growing to dangerous proportions. . . . It is imperative that the citizens of each land understand their national similarities and differences, place differences and antipathies in proper perspective, cherish precedents in cooperative action, coordinate for mutual advantage the factors of our economic life, respect our separate national cultures. . . . The educational institutions of the United States and Canada alike have a heavy responsibility for building in generation following generation the understanding and tolerance upon which good international relations rest. . . . There is evidence that educational agencies are not now discharging these responsibilities in adequate measure . . . ."

After reviewing the international influences and trends affecting education in the postwar period, the Committee "calls upon the educational forces of Canada and the United States to cooperate in insuring an adequate educational undergirding for the perpetuation of the international amity in North America which now exists. Increased and improved education about Canada in the United States and about the United States in Canada is possible and is desirable. It is a safeguard of future welfare. Such education must be realistic, not sentimental or propagandistic; it must be built on the assumption that mutual understanding and tolerance of differences are essential ingredients of enduring mutual respect."

The Committee regards education for good relations between Canada and the United States as one phase of a broader program of education for constructive participation in world affairs. The Committee in no way advocates a continental isolationism. At its Niagara Falls meeting it endorsed unanimously a proposal for es-

tablishing in the near future a joint United States-United Kingdom Committee on Education, similar in function and organization to the Canada-United States Committee. The Committee looked with favor upon close, three-way cooperation in educational and cultural matters among the educators of the United States, Canada, and Great Britain as an immediately feasible step in the development of strong, world-wide educational relations.

The Committee considered plans for and strongly recommended a proposed survey of textbooks and other teaching materials in history, geography, and the other social studies. The survey should analyze and evaluate these teaching materials and the courses of study in which they are used in the schools of both countries. It should describe what pupils in the United States are now taught about Canada and what pupils in Canada are now taught about the United States, and should lead to constructive suggestions for improved treatment of matters of mutual concern.

Teacher-education in the field of United States-Canada relations is regarded by the Committee as of pressing importance. The Committee is formulating plans for programs of teacher training, especially through summer schools, workshops, and travel designed to acquaint Canadian teachers with the United States and teachers from the United States with Canada. The Committee expects to issue a periodic newsletter to serve as a clearing house for information on activities and promising practices in the area of its interest. Individuals interested in securing the newsletter are invited to write to any member of the Executive Board of the Committee. The Committee will welcome information concerning school and college practices and plans in the study of Canada-United States relations.

Dr. E. H. Taylor, head of the department of mathematics at the Eastern Illinois State Teachers College at Charlestown, has retired after many years of distinguished service. Dr. Taylor has taught at Charlestown since the opening of the College in 1899.

Dr. Taylor is a charter member of The National Council of Teachers of Mathematics and has been active in its affairs. His many students and his large circle of friends will be glad to know that he is in the best of health and will devote his time to revising his textbooks. His many years as a superior type of teacher qualify him for this type of work.

Flight Experience in High School Aviation Courses Recommended by Six States

Flight experience to the extent of four hours in a dual-control airplane for each boy and girl in high school aviation classes has been recommended for the consideration of local schools by the State Departments of Education of Wisconsin, Illinois, Colorado, California, Connecticut and Pennsylvania. Each of these states has developed a comprehensive state plan for aviation education for the first twelve grades, and some have also included junior college and college programs.

The recommended aviation education programs vary considerably from state to state. For instance, Colorado is distinctive in recommending a separate course in Social Aviation for junior high schools which deals with the political, economic, and social implications of the Air Age. California, as would be expected, has a forward-looking program for junior colleges. The Connecticut report describes the college programs of that state in considerable detail and makes recommendations concerning them. Both Pennsylvania and Connecticut favor an elective aviation science course for junior and senior boys and girls in high schools which will have subject prerequisites and a level of difficulty adapted to their needs—so that at least 75% of all juniors and seniors may elect the course without an abnormal number of failures. Illinois is planning to embark on an extensive program of aviation-centered industrial arts and vocational courses. The Wisconsin plan, which was the first to be completed and published, emphasizes the objectives of a proposed high school aviation course and the relationships of the proposed laboratory flight experience to this course.

Some of the principles upon which all six of the state plans thus far completed agree are:

(a) A thorough program for infusing appropriate Air Age materials into all regular courses of study at all grade levels, in order to modernize and enrich the curriculum.

(b) Certain regular aviation courses, both in senior high schools and in colleges, to be continued and further developed as permanent parts of the postwar program in science education and its social significance.

(c) Four hours of flight experience for all high school aviation students, conceived as laboratory work in connection with classroom work in aviation, to be given under full insurance and liability protection for all parties concerned and to be organized to insure safety in every detail.

(d) Complete state and local responsibility for making aviation education an integral part of the state and local educational systems.

The Aviation Education Service of the Civil Aeronautics Administration has cooperated with the respective State Departments of Education in the development of all these state plans. The State Aeronautics Commission was co-sponsor of the conference which formulated the Pennsylvania plan.

Other groups interested in aviation education have also cooperated actively in the formulation of these comprehensive definitions of educational policies and proposed practices. There is no proposal of any Federal program of aviation education in any of these state reports, and there is no implication that there is to be Federal financial aid for any of the recommended programs. All organization, financing and control of these programs have been left to the state and local educational authorities except insofar as safety requirements of the Federal statutes must be met.

Publication of the reports have been completed in some of the states, while others are under way. Inquiries in regard to copies may be directed to the respective State Departments of Education or to the Aviation Education Service, CAA, Reference A-6, Department of Commerce Building, Washington, D. C.

THE WHITE HOUSE
WASHINGTON

August 28, 1944

TO THE PATRONS, STUDENTS AND TEACHERS OF AMERICAN SCHOOLS:

Upon the occasion of this, the twenty-fourth annual observance of American Education Week, I again call upon school patrons and citizens generally to visit their schools. Become better acquainted with those faithful servants of the nation's children and youth—the teachers.

Uphold their hands, acknowledge your appreciation, encourage them in their task of cultivating free men fit for a free world. For these teachers are the conservators of today's civilization and the architects of tomorrow's world of promised peace and progress. They serve within the very citadels of democracy, devotedly whether in war or in peace.

When victory on the fields of battle shall have been achieved, the work yet to be done through our schools will be enormous. I therefore call upon the teachers of America to continue without flagging their efforts to contribute through the schools to that final consummation which alone will make possible of fulfillment all plans of education for new tasks.

(Signed) FRANKLIN D. ROOSEVELT

"Education for New Tasks" is the theme for the 24th annual observance of American Education Week, November 5–11, 1944. This theme will be highly appropriate at a time when the end of the war in Europe is imminent if it has not already come.

We spare no expense to get people ready to win the war. Why? Because we know that only a trained people can win. Public sentiment would not tolerate for a moment a proposal to send any American boy into battle without the best of training under the best instructors and with the best equipment that money can buy.

Shall we do less to prepare our young people to win the battles of the peace? This is the most important question before the nation. There are, to be sure, immediate and pressing issues with reference to partial reconversion of industry, to the relaxation of government controls, to evidences of intolerance, and many others.

But ultimately we shall succeed in dealing with the peacetime problems ahead in direct proportion to the manner in which our human resources are prepared to deal with them. Now is the time when we must plan an educational program which will help the youth of today to win the peace.

American Education Week comes at an opportune time to stress the crucial role the schools can play in the nation's future. Plan now for an effective observance. Write the National Education Association, 1201 Sixteenth Street, N.W., Washington 6, D. C., for a list of helpful materials such as poster, manual, leaflets, stickers, plays, radio scripts, and other materials.

Announcement has just been made of the formation of a new society for mathematical research, the Duodecimal Society of America, established as a voluntary, nonprofit organization incorporated in New York state. The purpose of the Society, as stated in its constitution, is "to conduct research and education of the public in mathematical science, with particular relation to the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science."

"We count by tens for no better reason than that we happen to have ten fingers," says F. Emerson Andrews, president of the Society. "Philosophers and mathematicians have long agreed that twelve is a better and more efficient number base than ten; its actual use has been advocated at various times, notably by Herbert Spencer, the British philosopher, and Isaac Pitman, inventor of an early system of shorthand.

In the past few years, interest has grown in this old idea. A group of experimenters have been trying it out in both practical and theoretical problems—from helping the U. S. Army transport service figure cargo cubages to working with factorials. For some kinds of problems, the savings introduced by dozen-counting are no less than astonishing. The growing number of experimenters and the importance of their findings made a common center for information and further research a necessity, and the Duodecimal Society of America was formed."

According to the literature of the society, counting by dozens can be learned by anyone in the space of about half an hour. Two new numerals are required—*X* to represent ten, and *E* for eleven. Decimals are replaced by duodecimals, so that .4 means four-twelfths, and is a perfect third—a fraction for which there is no accurate decimal equivalent.

Officers of the Society are George S. Terry of Hingham, Mass., chairman of the board; F. Emerson Andrews of New York City, president; F. Howard Seely of Oakland, Calif., vice-president; and Ralph H. Beard of New York City, secretary and treasurer. The Society is located at 20 Carlton Place, Staten Island 4, New York.

WAR BOND MUSICAL SHOW BECOMES NATIONWIDE HIT

Since early Spring the new Treasury War Bond musical show "Figure It Out" has been produced in at least twelve states. In some instances it has been done simply with only 30 or 40 students during an assembly period, while other schools have made it an extravaganza for a full evening's entertainment with a cast of several hundred.

With catchy tunes and amusing dialogue "Figure It Out" dramatizes the high cost of living and how the individual can fight that rise through buying War Bonds and Stamps. Such songs as "Double Duty Dollar" and "I'm an Inflationary Dollar" give pertinent advice to the audience to save more.

That means that "Figure It Out" serves a double purpose. While it is providing real entertainment, it is bringing to the community the philosophy of the nation's War Finance program so that each citizen may "Figure It Out" for himself, and save accordingly.

Information about free materials for staging "Figure It Out" may be obtained from the Education Section, War Finance Division, Washington 25, D. C.

Schools Race Past $500 Million Goal for Over $600 Million in 1943–44 War Savings

This is better than 20% above the goal of $500,000,000 and early year-end estimates of $510,000,000.

"A half billion dollars and more is a *lot of money* for the school children to have saved for Stamps and Bonds," was the comment of President Roosevelt when he received the official 1943–44 Schools-at-War report inscribed on a Jap propeller tip shot down in the Pacific. The presentation was made in the White House library by a 13-year-old Virginia school boy who had helped to organize his school's monthly War Bond rallies, had raised the Schools-at-War flag each morning, and had himself earned enough money for three War Bonds since Christmas.

Previously, elaborate ceremonies had been held at Stout Field, Indianapolis, when the school-financed war equipment was accepted for the armed services. In a literal sense the Stout Field presentation became a Report to the Nation, for it was featured in a newsreel released in every theater during the last two weeks of the Fifth War Loan. The movie also included several shots made in the Indianapolis schools where students were shown buying and selling War Stamps in the classroom, pasting up their Stamp albums and working to earn money for more War Bonds.

The saving of American school children added up to more than $600,000,000 worth of trouble for the Axis—this school year.

At first the youngsters concentrated on jeeps and paid for 33,100 jumping jeeps at $1,165, 11,600 amphibious jeeps at $2,090, and 11,400 flying jeeps at $3,000. For good measure they threw in 7,690 parachutes, 6,170 life floats, and 5,190 motor scooters.

At the same time they ran campaigns by the thousands for field ambulances, sets of clothing and equipment, diving outfits, motorcycles, potato peeling machines, army mules, tanks, and walkie-talkies.

Not content with their own savings and the campaigns within their schools, the youngsters in the past year have made a concerted effort to take War Savings to the community. They have become educators for personal savings and frequently have taken over as efficient salesmen to follow up their lessons with practical applications.

During the Fourth War Loan, it is estimated that over $229,000,000 in War Bonds were sold through the schools. In many communities the boys and girls took entire charge of the Fourth and Fifth War Loan Drives.

As the school War Savings program enters its third year, the entire nation is looking to the schools for enthusiastic leadership.

November Meeting of the New Jersey Association

The Association of Mathematics Teachers of New Jersey will hold its eightieth regular meeting in the Roof Garden of the Hotel Pennsylvania, New York City on Friday, November 10, 1944. An all-day convention has been arranged and the following program set up:

I

10:00 to 10:20 — Business Meeting

10:20 to 10:30 — "Presentation of a Plan for a Sound System of State Support of Public Education."

Speaker provided by the State Assoc. NJEA.

II

10:30 to 11:15 — "Arithmetic—Teaching through Insight."
Dr. Fred L. Bedford
State Teachers College, Jersey City, New Jersey

11:15 to 12:00 — "Geometry—Teaching for Use."
Dr. Nathan Lazar
Mathematics Department, Midwood High School
Brooklyn, New York

III

12:30 — LUNCHEON

IV

1:45 — "What Is Ahead for High School Mathematics?"
Dr. Raleigh Schorling
Professor of Mathematics
and
Head of the Department of Mathematics
School of Education
University of Michigan, Ann Arbor, Michigan

Association Officers

President—J. Dwight Daugherty, Eastside High School, Paterson, N. J.

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OFFICIAL NOTICE BY EDWIN W. SCHREIBER, SECRETARY

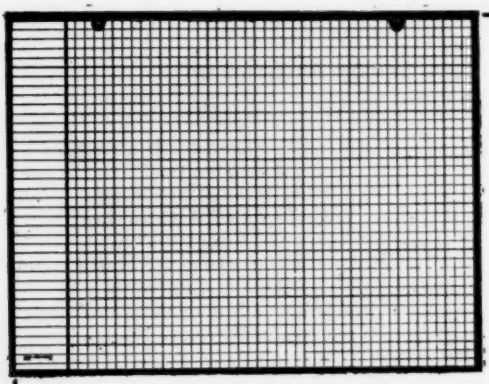
The National Council of Teachers of Mathematics

As SECRETARY of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place February 16, 1945.

At the Atlantic City meeting, Feb. 26, 1938, the Nominating Committee, consisting of the two most recent ex-presidents and the secretary as chairman (for this year: Mary A. Potter, Rolland R. Smith, and Edwin W. Schreiber), was instructed to prepare an official ballot naming two eligible candidates for each elective office, reserving a blank space for a third prospective candidate whose name may be written in by the voter. The officers to be elected are: Second Vice-President, 1945–1946, and three Directors, 1945–1947. The official ballot will be sent to members through the mail in January, 1945.

The periods of service of the officers of the National Council, from its organization in February, 1920 to the present time are printed on the following page.

EDWIN W. SCHREIBER, *Secretary*



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